1 Introduction

There is a growing consensus that trade liberalization can lead to substantial welfare gain to both origin and destination country. However, less attention has been devoted to examining the distributional impacts of trade. Specifically, who are the biggest winner at each country from trade liberalization? In terms of the welfare comparison between a developing and a developed country, another important question is that whether or not trade liberalization can uniformly shrink or widen the gap between the countries among all individuals? If not, this implies some individuals from the developing country may catch up while others may fall behind even further from those in the developed country. In this paper, we develop a quantitative trade model with heterogeneous agents to study these issues.

Several recent literature have aimed to embed non-homothetic CES preferences into the class of growth and new trade models. [1] Examples include Matsuyama (2015) and Comin et al. (2015). These studies focus on examining how changes in the preference structure affect the aggregate performance of the economy and the pattern of trade across countries. The benefit of having a non-homothetic preference is that the demand for every good does not have unitary income elasticity, and thus rich countries tend to export high-income elastic goods in the equilibrium. In this paper, we also introduce non-homothetic preference structure into the setting but with a major focus on the impact of trade liberalization at both sectoral and individual level. Similar to Matsuyama (2015), as a result of the "home market" effect documented in Krugman (1980), rich countries export high-income elastic goods because their demand composition is more skewed towards high-income elastic goods. In addition, we also introduce the endogenous labor supply decision, which tends to make income distribution become less dispersed than the human capital distribution if the more talented ones choose more leisure time.

In our model, individuals are heterogeneous in their human capital endowment. Countries possess different human capital distributions. Individual obtains utilities from final consumption and leisure time, while final consumption is an implicit non-homothetic CES aggregator of sectoral-level consumption. At a lower tier, sectoral consumption is a homo-

[1] The New trade theory is proposed in Krugman (1980), and developed by Melitz (2003) and Eaton and Kortum (2002). They usually feature homothetic preferences and monopolistic or perfect competition.

thetic CES aggregator of all the varieties available in the sector. The market structure at the production side of the economy is modeled as monopolistic competition. Each firm is the single producer of one variety. Free entry prevails in each sector.

The theoretical framework has the following distinctive features. First, by introducing heterogeneity in human capital endowment, we can define a human capital elasticity of sectoral consumption demand. The rank of the income elasticity over all the sectors are still maintained in the defined human capital elasticity, so the more talented ones proportionally demand more for the goods with higher income elasticity. Second, when we introduce the endogenous labor supply decision, the human capital elasticity can be further decomposed into an income elasticity and a human capital elasticity of labor supply. The latter is proven to be less than one with reasonable parameterization. Therefore, sectoral consumption tends to be less responsive to changes in human capital than in income. Moreover, we are able to show the human capital elasticity of final goods consumption tends to be decreasing with the human capital level, which hints a non-monotonic pattern in gains from trade among individuals. Third, when both economies open up, the more talented country demands more from the sector with the highest human capital elasticity. Due to the home market effect, more varieties will be produced and the sectoral price will tend to be lower, and thus the more talented country is more likely to become the net exporter of the sectoral varieties.

We calibrate the model into a two-country open economy, where the human capital distribution in the north country fist order stochastically dominates that in the south country. There are three sectors in the calibrated economy: agricultural, manufacturing and service sector. The major resources in the north country are calibrated to be allocated at the service sector, whereas the major resources in the south country are allocated at the agricultural sector. Therefore, north country is denoted to be a more talented and developed country, and the south is still a developing country. The benchmark results suggest that trade undoubtedly leads to welfare gains for all the individuals. In particular, in the south country the welfare gains decrease with human capital level, while in the north country the welfare gains exhibit a hump-shape pattern, where individuals close to the bottom of the human capital distribution collect the largest welfare gain. The hump-shape pattern can be attributed to the fact that the human capital elasticity of final goods consumption tends to be decreasing with the human capital level.

The quantitative results also suggest that individuals from the north country always enjoy higher welfare than those from the south country in the autarky economy. However, the least talented ones from the south country enjoy higher welfare than those from the north in the trade economy. As for whether trade widens or shrinks the utility gap between the two countries, we find that the utility gap shrinks when the human capital level is relatively low, and becomes widened at high human capital levels. The patterns are mainly driven by the home market effect. Price index in the service sector of north country and agricultural sector of south country declines the most in each country, respectively. The least talented and most talented ones constitute the major demand for agricultural and service goods, respectively. Therefore, the least talented ones in the south country benefit proportionally more than those in the north, which explains why they catch up in the trade economy. On the other hand, the most talented ones in the north country benefit proportionally more than those in the south, which suggests a widening pattern. Our findings also shed light on policy issues such as how trade protectionism can affect the welfare at both origin and destination countries.

To further evaluate the role of non-homothetic preferences and endogenous labor supply in driving the benchmark results, we perform several counter-factual exercises in which we either assign a CES preference structure or let the labor supply be exogenous. We summarize the main findings as follows: 1) When labor supply is exogenous, the gain from trade in the north country become increasing with the human capital percentiles as opposed to decreasing in the benchmark results. 2) Under the CES preference structure, the utility gap in the trade economy strictly decrease with the human capital level in comparison with the hump-shape pattern in the benchmark results. 3) Under both assumptions, individuals close to the bottom of the human capital distribution no longer obtain higher welfare than those in the north country. 4) The utility gap between the countries uniformly shrink from the autarky to the trade economy in both cases. All these results highlight the importance of both assumptions.

The mechanism is the following: when labor supply is exogenous, the human capital elasticity of sectoral consumption is equivalent to the income elasticity, and greater than

1 in the most income elastic sector. Therefore, the consumption ratio in it between an individual of high human capital to one with relatively lower human capital will be larger in the case of exogenous labor supply than the benchmark scenario. This could potentially attribute to the increasing pattern in gains from trade against human capital. Under the CES preference structure, the human capital elasticity of sectoral consumption is identical in each sector, and equivalent to the human capital elasticity of labor supply, which is less than one. Therefore, the more talented ones will no longer spend proportionally more than others in the most income elastic sector ranked in the benchmark scenario. When resource become more evenly allocated in the north country under the CES preference structure, these suggest the least talented ones in it may not be worse off than those in the south, as well as a uniformly shrinking utility gap from autarky to the trade economy.

Our model is built upon the class of new trade models, such as Krugman (1980), and multiple variations and extensions of Melitz (2003). However, these models typically assume that consumers have identical and homothetic preferences in all countries. As argued in Markusen (2013), such models imply that aggregate demand only depends on price indexes and aggregate income, and is independent of the distribution of income. We introduce non-homothetic preferences and heterogeneous individuals to an otherwise standard new trade model. The purpose is to evaluate the distributional impacts from changes in preference structure and trade liberalization. Our paper is most closely related to Matsuyama (2000), Matsuyama (2015) and Comin et al. (2015). Matsuyama (2015) focuses on how non-homothetic preference structure may affect the trade pattern between two countries that differ only in population size. Individuals are homogeneous in his paper, and it is purely theoretical; simplification assumptions are required to obtain more analytical results. Comin et al. (2015) embed non-homothetic preferences into a multi-sector growth model to generate non-homothetic Engel curves at all levels of development.Duarte and Restuccia (2010) use a model with non-homothetic preferences to study structural transformation, and argue that non-homothetic preference is one reason for labor reallocation across sectors. Both households and firms are representative in their work, and the economy is closed and the role of trade liberalization is absent.

The major contribution of this paper are summarized as follows. First, this theoretical framework is probably the first to embed non-homothetic CES preferences and heterogeneous individuals into a trade model with monopolistic competition. This framework allows us to explore the distributional impacts of trade liberalization while taking account non-unitary income elasticity. Second, we highlight the importance of non-homothetic preference and endogenous labor supply in both analytical and quantitative results. Including leisure into the utility function makes income distribution less dispersed than human capital distribution. It also adds an extra term to human capital elasticity, which is thus lower than conventionally defined income elasticity. Under the CES preference structure, the human capital elasticity become identical over all the sectors. The quantitative results also depend crucially on both assumptions. In addition, the assumption in heterogeneity as well as the endogenous labor supply greatly complicate the analysis compared with Matsuyama (2015), yet we are still able to obtain most analytical results even under a more general parameter space. Third, the sectoral price index may response asymmetrically to trade due to different market sizes in each country. In addition, the trade-off between leisure and consumption varies over individuals of different human capital levels. Therefore, it remains as quantitative questions on who will collect the largest welfare gain from trade and how the utility gaps between the two countries change from autarky to the trade economy. Therefore, we go beyond Matsuyama (2015) by providing detailed quantitative analysis to address those issues from aggregate, sectoral, and individual-level, respectively. The findings suggest trade liberalization may not necessarily lead to a convergent outcome in welfare between a developed and a developing country. The least talented and thus the poorest one may catch up those in the developed country, but the gap among the richest ones may be amplified.

The rest of this paper is organized as follows. Section 2 describes the model setting. We perform quantitative analysis in Section 3. Our conclusion is offered in Section 4.

2 Model

The theoretical framework is an extension of Matsuyama (2015), with heterogeneous workers and endogenous labor supply decision. Moreover, we allow more flexible assumptions in the parameterization, but the analytical results still remain tractable. We first describe the model setting in the case of a closed economy. We then characterize the optimization problem for both individuals and firms. The equilibrium is then defined and some analytical properties are discussed. Finally, we open up the economy and examine how the patterns of trade are endogenously determined within the framework.

The economy is populated with a mass N of workers. Workers are heterogeneous in term of their human capital endowment, which is assumed to follow a certain distribution function $G(\cdot)$. There are a finite number $S > 1$ of sectors in the economy.

2.1 Preferences and Individual Optimization Problem

Individual workers obtain utilities from final consumption as well as leisure. Specifically, the utility function for a worker with human capital h takes the following specific form:

$$U(h) = \frac{C(h)^{1-\sigma}}{1-\sigma} + \theta \frac{\ell(h)^{1-\sigma}}{1-\sigma},$$

where $C(h)$ denotes the final consumption and $\ell(h) \in (0,1)$ is the leisure time, where we normalize the time endowment for each worker to be 1. $\theta > 0$ governs the preference of leisure against final consumption.

We follow Matsuyama (2015) by assuming that final consumption is an implicit non-homothetic CES aggregator of sector-level consumption. Specifically, a worker with human capital h has final consumption $C(h)$ defined as follows:

$$\sum_{s=1}^{S} C\left(h\right)^{\frac{\epsilon(s)-\eta}{\eta}} C_s\left(h\right)^{\frac{\eta-1}{\eta}} = 1, \tag{1}$$

where $C_s(h)$ denotes sector-level consumption. When $\epsilon(s) = 1$, the functional form above can be reduced to standard homothetic CES preference. We formally characterize the optimization problem for a representative worker of human capital h as follows:

$$\max_{C_s(h),\ell(h)} \quad \frac{C(h)^{1-\gamma}}{1-\gamma} + \theta \frac{\ell(h)^{1-\gamma}}{1-\gamma}$$
$$\text{s.t.} \quad \sum_{s=1}^{S} C\left(h\right)^{\frac{\epsilon(s)-\eta}{\eta}} C_s\left(h\right)^{\frac{\eta-1}{\eta}} = 1,$$
$$\sum_{s=1}^{S} C_s(h) p_s = E(h) \equiv wh(1-\ell(h)).$$

The last line describes an individual's budget constraint, and p_s is the price level in sector s. w is the exogenous wage rate, and $h(1-\ell(h))$ is the efficiency labor supply. We solve the optimization problem above for optimal final and sectoral consumption as well as labor supply decision in the Appendix A.1. The sectoral consumption can be expressed as:

$$C_s(h) = \frac{E(h)^{\eta} C(h)^{\epsilon(s)-\eta}}{P_s^{\eta}}.$$

The income of the sectoral-level consumption demand $C_s(h)$ can thus be expressed as:

$$\frac{\partial \ln C_s\left(h\right)}{\partial \ln E(h)} = (\epsilon(s)-\eta)\frac{\partial \ln C(h)}{\partial \ln E(h)} + \eta, \tag{2}$$

$$= \left(\varepsilon\left(s\right)-\eta\right) \frac{\sum_{t=1}^{S} P_t^{1-} C\left(h\right)^{\epsilon(t)-\eta}}{\sum_{t=1}^{S} \frac{\epsilon(t)-\eta}{1-\eta} P_t^{1-\eta} C\left(h\right)^{\epsilon(t)-\eta}} + \eta. \tag{3}$$

A number of features stand out from our setting. First, if we rank sectors according to the value of $\epsilon(s)$ in ascending order, then sectors of a higher index have a higher value of ϵ. We restrict the analysis to the case where $0 < \epsilon(s) < \eta$ and $\eta > 1$. It is straightforward to show that the wage elasticity of the sectoral-level consumption increases with the sector index. Second, the first part of Equation (3) can be shown to be less than $1-\eta$ in the sector with the lowest index, and larger than $1-\eta$ in the sector with the highest index. Specifically, we have

$$\frac{\partial \ln C_1\left(h\right)}{\partial \ln E(h)} < 1 \quad \text{and} \quad \frac{\partial \ln C_S\left(h\right)}{\partial \ln E(h)} > 1.$$

The implication is that sector 1 is the least wage-elastic sector, with elasticity less than one, and sector S is the most wage-elastic sector, with elasticity greater than one. Moreover, in any sector s, wage elasticity positively depends on the individual human capital level. That is, sectoral consumption is more elastic for individuals with higher human capital.

We can also show that the price elasticity of sectoral-level consumption is independent on individual's human capital level and equal to η:

$$\frac{\partial \ln C_s(h)}{\partial \ln P_s} = \eta.$$

At a lower tier, sectoral consumption is a homothetic CES aggregator of all the varieties available in the sector. Specifically, the consumption level in sector s for an individual with human capital h, $C_s(h)$, can be expressed as:

$$C_s(h) = \left[\int_{v \in \Omega_s} c_s(v,h)^{\frac{\sigma-1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}},$$

where $c_s(v,h)$ denotes the consumption level for variety v within sector s for an individual with human capital h, and σ is the elasticity of substitution among all the varieties in sector s.

Conditional on the sectoral consumption demand $C_s(h)$ for an individual with human capital h, the demand for a single variety within sector s can thus be obtained by solving the following expenditure minimization problem:

$$\begin{aligned} &\min_{D_s(v,h)} \int_{v \in \Omega_s} p_s(v) c_s(v,h) dv \\ &\text{s.t.} \quad \left[\int_{v \in \Omega_s} c_s(v,h)^{\frac{\sigma-1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}} = C_s(h), \end{aligned}$$

where $p_s(v)$ is the price level for variety v in sector s. The solution yields:

$$c_s(v,h) = \frac{C_s(h)}{\int_{v \in \Omega_s} p_s(v)^{1-\sigma} dv} p_s(v)^{-\sigma}$$

We define the sectoral price index P_s as: [2]

$$P_s = \left[\int_{v \in \Omega_s} p_s(v)^{1-\sigma} dv \right]^{\frac{1}{1-\sigma}}.$$

2.2 Production

Production within each sector is characterized as monopolistic competition. Each firm is the single producer of a variety. Following Krugman (1980), firms are all identical within each sector. Labor is the only input of production. To produce one unit of variety in sector s requires ψ_s unit of labor. Free entry into each sector is assumed, and ϕ_s labor is required as the entry cost into sector s. Solving a monopoly firm's profit maximization problem by taking the individual worker's demand function as given yields:

$$p_s(v) = w\psi_s \frac{\sigma}{\sigma - 1}.$$

Free entry condition implies that each individual firm's sales revenue shall equal the sum of production and entry costs. That is:

$$p_s(v) D_s(v) - \psi_s D_s(v) w = w\phi_s.$$

$D_s(v)$ is the total demand for variety v, which is the sum of all individual workers' demands:

$$D_s(v) = N \int c_s(v, h) dG(h).$$

The quantity being produced for each variety in sector s can be solved from the free-entry

[2] The sector-level price level P_s is the cost to obtain one unit of sectoral output, which is essentially the solution to the following expenditure minimization problem:

$$\min \int_{v \in \Omega_s} p_s(v) c_s(v, h) dv$$
$$\text{s.t.} \quad \left[\int_{v \in \Omega_s} c_s(v, h)^{\frac{\sigma-1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}} = 1,$$

condition as:

$$D_s(v) = \frac{(\sigma-1)\phi_s}{\psi_s}.$$

Total sectoral sales revenue thus equals $I_s w\sigma\phi_s$, where I_s denotes the number of firms in sector s. By equalizing above with the total expenditure on goods from sector s, we have:

$$I_s w\sigma\phi_s = N \int P_s C_s(h) dG(h).$$

The number of firms in sector s can thus be pinned down from the equation above.

2.3 Equilibrium Characterization

We first summarize the equilibrium definition, then proceed to characterize some analytical properties, and define the human capital elasticity in this subsection.

Definition Given $\{\phi_s, \psi_s\}$ and the wage rate w, the competitive equilibrium consists of a series of prices $\{p_s(v), P_s\}$, consumption quantities $\{C_s(h), c_s(v,h)\}$, the number of firms in each sector I_s, and the labor supply decision made by each individual $\ell(h)$, such that the following conditions hold:

1. Given $\{p_s(v), P_s\}$, each individual worker maximizes his utility by choosing $C_s(h), c_s(v,h)$ and $\ell(h)$.

2. Given the aggregate demand function from individual workers, each monopoly firm chooses the price of the variety to maximize its profit.

3. Free entry condition holds.

4. Labor market clears.

In Appendix A.1 we have obtained that the optimal leisure choice satisfies:

$$l\left(h\right)^{\gamma} = \frac{\theta E\left(h\right)^{\eta-1}\left(1-\ell\left(h\right)\right)}{1-\eta} \sum_{k} \left(\epsilon\left(k\right)-\eta\right) p_k^{1-\eta} C\left(h\right)^{\left(\varepsilon(k)-\eta-1+\gamma\right)}. \tag{4}$$

We further explore the relation between human capital endowment and labor supply as well as the final goods consumption in the following proposition.

Proposition 1 *In the equilibrium, when* $1 < \varepsilon(s) < \eta$*, the following arguments hold:*

(i) *the optimal final goods consumption is increasing with human capital, while the optimal labor supply is decreasing with human capital.*

(ii) *The human capital elasticity of labor supply is between 0 and 1.*

(iii) *The human capital elasticity of final goods consumption is decreasing with human capital level.*

Proof *See the Appendix.*

The proposition above suggests that individuals with higher human capital will enjoy higher final goods consumption as well as more leisure time, which certainly leads to a higher utility. Doubling the human capital level will lead to a less than 100-percent growth in the leisure time. Moreover, if an individual's human capital is 10-percent higher than others, the ratio in final goods consumption between them is larger at the lower end of the human capital distribution.

Different from Matsuyama (2015) and others, income is no longer exogenous in our model, and instead depends on the labor supply decision made by the individual of different human capital levels. Therefore, it is suitable to define a human capital elasticity of sectoral consumption, which can be expressed as follows:

$$\frac{\partial \ln C_s(h)}{\partial \ln h} = \frac{\partial \ln C_s(h)}{\partial \ln E(h)} \frac{\partial \ln E(h)}{\partial \ln h} \tag{5}$$

$$= \underbrace{\left\{ (\varepsilon(s) - \eta) \frac{\sum_{s=1}^{S} P_s^{1-\eta} C(h)^{\epsilon(s)-\eta}}{\sum_{s=1}^{S} \frac{\epsilon(s)-\eta}{1-\eta} P_s^{1-\eta} C(h)^{\epsilon(s)-\eta}} + \eta \right\}}_{\text{income elasticity}} * \underbrace{\left\{ 1 - \frac{\ell(h)}{1-\ell(h)} \frac{\partial \ln \ell(h)}{\partial \ln h} \right\}}_{\text{human capital elasticity of labor supply}} . \tag{6}$$

We can decompose human capital elasticity defined above into two parts: an income elasticity and a human capital elasticity of labor supply. The role of exogenous labor supply and non-homothetic preference structure can both be highlighted from the expression above. When the preference structure becomes CES, the human capital elasticity is equivalent to labor supply elasticity, which is strictly less than 1 according to Proposition **??**. This implies that individuals with higher human capital may not spend proportionally more than others in goods produced from any sector. In the case of exogenous labor supply, the human capital elasticity will be identical to the income elasticity, so the most talented ones will spend proportionally more than others in the sectors of the highest index. This suggests that consumption demand is less responsive to changes in human capital in the case of endogenous labor supply compared with exogenous labor supply. We will emphasize both assumptions are quantitatively important in evaluating individual's welfare gain from trade in Section 3.3.

2.4 Trade Equilibrium and Patterns of Trade

We apply the setting to an open economy with two countries, country 1 and 2. In the purpose of obtaining more analytical results, we assume the two countries are identical in every aspect except for the initial human capital distribution. We will relax this assumption in the quantitative exercises. We let the distribution in country 2 first-order stochastically dominate that of country 1. Therefore, country 2 is denoted as a more talented economy, as it comprises more individuals with higher human capital.

Standard iceberg trade cost assumption applies. To deliver 1 unit of goods from country j to i, $\tau_{ij} > 1$ units of goods need to be shipped. Country $j's$ demand for a variety produced in sector s of country i can thus be solved as:

$$D_s^{ij}(v) = N^j \frac{\int_{\underline{h}}^{\overline{h}} E^j(h)^{\eta} C^j(h)^{\varepsilon^j(s)-\eta} dG(h)}{P_s^{j\eta-\sigma}} p_s^{ij}(v)^{-\sigma}.$$

The price for a variety sold to country j from sector s of country i is thus:

$$p_s^{ij}(v) = \tau_{ij} w^i \psi_s^i \frac{\sigma}{\sigma-1}.$$

Moreover, the free-entry condition in country j implies:

$$\sum_i \left[p_s^{ji}(v)\, q_s^{ji}(v) - \tau_{ji}\psi_s^j q_s^j(v)\, w^j\right] = w^j \phi_s^j.$$

In a closed economy, the wage rate is assumed to be exogenous. In the case of an open economy, the relative wage between the two countries will be pinned down by the trade balance in each country:

$$w^j L^j = \sum_{i=1}^{J} \frac{\sum_s V_s^j D_s^{ji}(v)\, p_s^{ji}(v)}{\sum_{j=1}^{J}\sum_s V_s^j D_s^{ji}(v)\, p_s^{ji}(v)} w^i L^i, \quad j \in \{1,2\}$$

The net export value in sector s of country i is the value of goods sold to country j minus the value of goods purchased from country j, which can be expressed as

$$NX_s^i = V_s^i D_s^{ij}(v) p_s^{ij}(v) - V_s^j D_s^{ji}(v) p_s^{ji}(v). \tag{7}$$

We denote m_s^i to be the expenditure share on sector s in country i, and L_i^e to be the total efficiency labor supply in country i. These can be expressed as:

$$m_s^i = \frac{\int E_s^i(h)\, dG^i(h)}{\int E^i(h)\, dG^i(h)} = \frac{\int E^i(h)^{\eta}\, C^i(h)^{\varepsilon(s)-\eta}\, (P_s^i)^{1-\eta} dG^i(h)}{\int E^i(h)\, dG^i(h)}, \tag{8}$$

$$L_i^e = N_i \int h(1-\ell(h)) dG^i(h). \tag{9}$$

In the Appendix A.3, we show in detail that the general formula for the net export value in country i can be expressed as:

$$NX_s^i = \rho L_j^e w^j \left\{ \frac{m_s^i \omega}{\omega^\sigma - \rho} \frac{L_i^e}{L_j^e} - \frac{\omega^\sigma m_s^j}{1-\rho\omega^\sigma} \right\}, \tag{10}$$

where $\omega = \frac{w^i}{w^j}$ and $\rho = \tau_{ij}^{1-\sigma}$.

When there are no sectoral differences in the labor requirement and entry cost, the expression above can be further reduced to the one in Matsuyama (2015):

$$NX_s^i = \frac{\rho L_i^e w^i}{\omega^\sigma - \rho}\left(m_s^i - m_s^j\right)$$

By evaluating equation 10, it is straightforward to demonstrate that changes in the following variables can contribute to a higher net export value:

- higher m_s^i or lower m_s^j;
- lower trade cost τ;
- higher ratio in the efficiency labor supply L_i^e/L_j^e.

Due to the non-homothetic preference structure, individuals with higher human capital tend to proportionally spend more on sectors with a higher index. Moreover, together with the home market effects proposed in Krugman (1980), these results imply that the more talented country tends to be a net exporter of goods from the sector with higher index.

3 Quantitative Analysis

We perform quantitative analysis in this section. We restrict the analysis to a two-country setting, where one corresponds to a developed country and the other is a developing country. The purpose is to quantify the effects of trade liberalization at the aggregate, sectoral and individual level. In particular, we focus on the distributional impacts of trade by examining how the gains from trade vary over individual's human capital level, and whether individuals from the developed country is always better off than those in the developing country. In addition, we are also keen to understand whether trade liberalization tends to widen or shrink the welfare gap between the two countries. Both non-homothetic preference structure and endogenous labor supply are the two key assumptions made in the paper. In order to evaluate the quantitative importance of both assumptions, we perform several counter-factual exercises by either assign a CES preference structure or let the labor supply be exogenous. In addition, we also examine the role of risk aversion in the equilibrium outcome.

3.1 Parameterization

We calibrate the model into a two-country open economy setting. We let human capital distribution in each country j follow a Pareto distribution defined over $[\underline{\mathrm{h}}, \infty)$:

$$G(h) = 1 - (\frac{\underline{h}}{h})^{\alpha_j}.$$

The minimum human capital level ($\underline{h}$) is set to be 1 in both countries. The two countries only differ in the skewness of the distribution governed by $\alpha_j > 0$. We let the number of sectors be three. The data counterpart refers to the the agriculture, manufacturing and service sector, respectively. We normalize the population size to be 1 in each country so that the differences in market size are only caused by differences in the human capital stock. γ governs the risk-aversion coefficient, and we follow the literature in setting it at 2. σ captures the sectoral-level price elasticity. We assume σ is larger than η to mimic the fact that differentiated goods are closer substitutes within each sector than across sectors. We let σ equal 6 and η equal 4, so they are consistent with the range commonly used in the literature. We normalize the labor requirement in the manufacturing sector ψ_{2j} to be 1 in each country j.

The remaining parameters include $\{\psi_{11}, \psi_{31}, \psi_{12}, \psi_{32}, \phi_{11}, \phi_{21}, \phi_{31}, \phi_{12}, \phi_{22}, \phi_{32}, \epsilon_1, \epsilon_2, \epsilon_3, \alpha, \theta\}$. We jointly calibrate them to simulate a hypothetical North-South economy similar to the definition in Grossman and Helpman(1991). North is a developed economy with completed structural transformation, and thus a large fraction of employment and firms belong to the service sector. In contrast, South is still a developing economy that heavily relies on its agricultural sector. ψ_{11} and ψ_{31} are the unit labor requirement for the agricultural and service sector in south country, respectively. We calibrate them to match a 50-percent employment share in the agricultural sector, and 20-percent employment share in the service sector. ϕ_{11} and ϕ_{31} are the entry costs in term of labor requirement in the agricultural and service sectors of south country, respectively. Similarly, we calibrate them to match a 50-percent of firms in the agricultural sector, and a 20-percent of firms in the service sector. As for ϕ_{12} and ϕ_{22}, we calibrate them so that the entry costs roughly take 10-percent of the sales revenue in each country. Similar to the calibration strategy in the south economy, ψ_{12} and ψ_{32} are calibrated to match a 20-percent and 50-percent employment share in the agricultural and service sector of north economy, respectively. ϕ_{12} and ϕ_{32} are chosen to match a 10-percent and 50-percent of firms in the agricultural and service sector.

As shown before, $\epsilon_i (i = 1, 2, 3)$ reflects the income elasticity of sector i. We follow Markusen (2013) to target an income elasticity of 0.65 in the agriculture sector, 1.0 in manufacturing sector, and 1.7 in service sector. α is the shape parameter of the Pareto distribution. We let $\alpha_2 < \alpha_1$ so that the human capital distribution of north country first-order stochastically dominates that of south, which implies north is a more talented country. We calibrate α_1 to match a ratio of 3.3 between the 90th and 10th percentile of the income distribution in the south country[3]. We set the shape parameter (α) in the north country to be 20 percent lower than that in the south country. θ controls the preference for consumption against leisure, and is chosen to match a 1/3 fraction of leisure time on average among all individuals. Finally, we calibrate the iceberg trade cost parameter τ to match a 20-percent trade share in the south country. We summarize the calibrated parameter values in Table C.1.

3.2 The Role of Trade Liberalization

Given the benchmark parameterization, we numerically solve the model. The algorithm is briefly characterized as follows: we first obtain demand at the upper tier: given the wage rate and the sectoral prices, we solve for sectoral consumption and leisure choice. At the lower tier, given the sectoral consumption, we solve for demand against each variety. On the production side, we pin down firm distribution across sectors using free-entry conditions. Finally, market clearing conditions pin down wages and sectoral prices. Once we obtain the benchmark results, we proceed to solve for the equilibrium in an autarky economy by setting the ice-burg trade cost parameter τ to a sufficiently high level. In the case of autarky, we also normalize the wage rate in both countries to be 1. We report the aggregate results for both benchmark and autarky equilibrium in Table C.2.

GDP in each country is essentially the total labor income since the free entry condition drives the business profits in each sector to zero. We define the welfare in each country as the

[3]The data source is from China Household Finance Survey and author's own computation

mean utility level that agents can obtain. We also follow the convention to define trade share as the ratio of total exports to the home country's GDP. Our results suggest that in both benchmark and autarky economy, North country enjoys a higher GDP as well as welfare level than the South country. When we measure income distribution using either Gini coefficient or the 90-10 income ratio, income tends to be more dispersed in the North than in the South, and this is likely due to a more dispersed initial human capital distribution in the North. In the benchmark economy, the South country exports a larger fraction of its GDP to North than vice versa. This is driven by a larger market size of north country, and thus a larger demand for goods produced in the south.

In the event of trade liberalization, the average welfare level are improved by 1.9 and 1.46 percent in South and North country, respectively. In contrast, GDP in both countries become relatively lowered. This is mainly because trade lowers the price index in each country, and thus agents get to reduce the labor supply and allocate more time to leisure. As a result, the nominal GDP falls.

We further examine the role of trade liberalization by comparing the equilibrium outcome at the sectoral level. The results are reported in Table C.3. The distribution of both employment and the number of firms in the benchmark results are targeted in the calibration exercise. From the autarky to the trade economy, a large fraction of workers and firms in the south country are reallocated from sector 3 to sector 1, while resources in sector 2 barely change. In contrast, in the north country resources flow in the opposite direction from sector 1 to sector 3. The results are not surprising. sector 1 incurs the highest expenditure share in the south country, while sector 3 in the north country attracts the highest expenditure. They thus induce the largest firm and employment inflow due to the classical "Home market" effect documented in Krugman (1980). This can also be reflected in changes of sectoral price index. In the south country, sector 1 has seen the sharpest decline in the price index at about 3.98 percent, followed by 2.7 percent decline in the price index of sector 2, while the price index in sector 3 has almost remained unchanged due to trade. In contrast, in the north country the price index of sector 3 has declined by 2.77 percent, while the price index at the other two sectors have almost stayed at the same level from autarky to trade. We define the sectoral net exports as the difference from exports to imports in terms of value at each sector. Our results suggest that south country is the major exporter of goods produced in sector 1, and it also runs a slightly positive trade surplus in sector 2, while the north country is the major exporter of goods produced in sector 3. Overall, trade balance is maintained at each country.

We have reported how trade liberalization affects the average welfare level in each country in Table C.2. We further discuss the distributional impacts of trade in Figure C.1. In panel (a) of Figure C.1, we have plotted the percentage change in utilities from autarky to the trade economy for individuals at different human capital percentiles in both countries. Undoubtedly, trade leads to a welfare gain for all the individuals, and the magnitude of the gain is larger in the south than the north country. Moreover, in the south country the welfare gain decreases with human capital level: individuals with the lowest human capital level enjoys the highest welfare gain at a level slightly above 2 percent, while individuals at the top of the human capital distribution receive the lowest welfare gain at around 1 percent. In the north country, there exhibits a hump-shape pattern in individual's welfare gain: those individuals whose human capital belong to the bottom 5th percentile of the distribution collects the largest gain from trade at around 1.5 percent. The most talented group of individuals receive the smallest gain at around 0.9 percent.

If we intend to address the issue that whether trade liberalization widens or reduces the welfare gap between the countries, it is not very fair to compare individuals at the same percentile of the human capital distribution from the two countries, because the distribution itself is different in the two countries. Therefore, in the following exercise, we restrict the analysis to a group of individuals of the same human capital levels from both countries. Panel (b) of Figure C.1 plots the position of each selected human capital level in the corresponding distribution function. It suggests that the selected human capitals have covered almost the entire human distribution in both countries. The hump shape pattern of welfare gain in the north country is likely due to the following: individuals at a relatively lower percentile of the distribution are the major consumers of goods produced in both sector 1 and 2, and thus they may benefit most from the sharp price decline in sector 1 and 2 in the south country due to trade liberalization.

In the panel (c) and (d) of Figure C.1, we report the utility gap between the north and south country for individuals of the same human capital level in the trade and autarky economy, respectively. Utility gap is simply defined as the level difference between the two countries. A positive number in the figure implies that individuals from the north country enjoy than those from the south country. The results show that the utility gap tends to be positive in the autarky economy. This implies that individuals from the north country always receive a higher utility than individuals of the same human capital level from the south country. The utility gap peaks at the human capital level close to 2, then gradually declines and reach the lowest level when human capital reaches the maximum level of 15. The highest and lowest utility gap takes approximately 1.08 and 0.4 percent of the average welfare level in the south country. When both economies open up, the utility gap still remains positive at most of the human capital levels except for those at the bottom of the distribution. This suggests that those least talented individuals from the south country indeed catch up those at the north country in terms of the welfare. The utility gap also peaks at a relatively higher human capital compared with the autarky economy. The hump shape pattern of the utility gap is likely due to the property that human capital elasticity of final goods consumption is decreasing with the human capital level documented in Proposition 1. As the human capital marginally increase at the lower end of the distribution, the surges in utilities may reach the highest level at both countries regardless of the autarky or the trade scenario, and this may result in the largest utility gap between the countries.

As for whether the utility gap between the countries become widened or shrinked, in the panel (e) of Figure C.1, we plot the net differences in the utility gap from the autarky to the trade economy, which is essentially the difference in the y-axis value from panel (d) to panel (c). Therefore, a negative number in the figure implies a shrinking gap, while a positive number refers to a widening gap. Our results show that the utility gap are shrinked when the human capital level is relatively low, and become widened at high human capital levels. Despite trade liberalization leads to an overall welfare gain to all the individuals, it tends to benefit proportionally more for those least talented individuals in the south country, and those most talented ones from the north country. The mechanism is straightforward: south country is a net exporter of sectors 1 and 2, while north country is a net exporter of sector 3. Trade liberalization proportionally induces more firm entries into sector 1 of south and sector 3 of north country, which lowers the price index in the specific sector proportionally more than other sectors. Due to the non-homothetic preference structure, the least talented individuals in the south country consume proportionally more than others in goods from sector 1, and thus they also benefit more from trade than others. Similarly, the most talented ones in the north country constitute the major demand for goods from sector 3, and thus they also enjoy proportionally larger welfare gain from trade than others. The non-homothetic preference structure is crucial in driving the pattern described above. We will illustrate in the next section that given a CES preference structure, utility gaps are uniformly shrinked from autarky to the trade economy.

3.3 Counterfactual Exercises

To further evaluate the role of non-homothetic preference structure, endogenous labor supply decision, as well as the risk-aversion attitude in driving outcomes at the aggregate, sectoral and individual levels, we perform several counter factual exercises in this subsection by either setting preferences to be homothetic, or making the labor supply be exogenous, or letting the individuals become more risk-averse.

In Table C.4 and Table C.5 we report the impacts of trade liberalization under a CES preference structure at the aggregate and sectoral level, respectively. We let ε take a simple average of the ε_s specified in the benchmark parameterization. Similar to the benchmark results, trade tends to lower the nominal GDP, increases the average welfare level as well as widens the income inequality at both countries. At the sectoral level, trade expands sector 1 in the south country at a cost of shrinking sector 2 and 3. Similarly, trade enlarges sector 2 and 3 in the north country. The difference between the benchmark and the CES preference structure is that resources are more evenly allocated in the north country. This is because human capital elasticity is indentical in all the sectors under the CES preference, and is no longer larger than 1 in sector 3. When more talented ones do not spend proportionally more than others in sector 3, sector 3 will no longer play a dominant role in the north country.

In Table C.6 and Table C.7 we report the aggregate and sectoral impacts of trade when the labor supply is exogenous and normalized to be 1. When labor supply decision is absent, individual's income is simply the wage rate multiplied by the individual's human capital level. Therefore, the income distribution will be identical to the human capital distribution, it thus remains unchanged from the autarky to the trade economy. Moreover, in the autarky equilibrium, wages in both countries are set to be 1, while in the trade equilibrium, we normalize wage rate in the north country to be 1. Therefore, it is a mechanical result that GDP remains unchanged in the north country when the economy opens up. In addition, welfare still improves in both countries in the trade equilibrium due to a lower price index at each sector. At the sectoral level, resources flow from sector 3 to sector 1 and 2 in the south country, while the opposite direction of flow takes place in the north country. In the trade equilibrium, the south country becomes the net exporter of goods produced in sector 1 and sector 2, while the north country is the net exporter of goods produced in sector 3.

In Table C.8 and Table C.7 we examine the role of risk aversion attitudes at both the aggregate and sectoral level. We increase the risk aversion coefficient σ to 3.0. When individuals become more risk-averse, they tend to reduce the labor supply, and thus the GDP levels at both countries become lower than the benchmark outcomes. Nevertheless, the impacts of trade when individuals become more risk averse remain qualitatively the same as the benchmark results.

We focus on the comparison of individual results across different scenarios in the following. In Figure C.2 we plot the percentage change in welfare from the autarky to the trade economy at both countries for all the four cases. Trade leads to welfare gain for all the individuals in all the cases. In a CES preference structure, welfare gain decreases with the human percentiles in the south country. However, unlike the benchmark results where the welfare gain peaks at bottom 5th percentile in the north country, it is also monotonically decreasing with the percentiles under the CES preference structure. The magnitude of the welfare gain is larger in the benchmark results for both countries than that under the CES preference structure. Moreover, it appears that the welfare gain tends to converge to the same percentage level as the percentile increases in the benchmark case. However, a divergent pattern is observed under the CES preference structure. This is likely due to the constant human capital elasticity of final goods consumption shown in Appendix A.2. When individuals become more risk averse, welfare gain monotonically decreases with the percentiles in both countries, and the welfare gain is larger than that in the benchmark results.

When labor supply becomes exogenous, our results suggest that all individuals obtain larger welfare gain than the case of endogenous labor supply. This is likely due to a larger market size reflected as higher total nominal income at both countries. Interestingly, the welfare gain is shown to be decreasing with percentiles in the south country but increasing with percentiles in the north country. If the labor supply is normalized to be 1 for all the individuals, the income differences in each country are completely reflected by the human capital differences, whereas in the case of endogenous labor supply, the income gaps tend to shrink by having more talented ones choose more leisure time. In addition, we have shown in the previous section that the human capital elasticity of consumption in sector 3 is lower in the case of endogenous than exogenous labor supply, and the latter one is shown to be strictly larger than 1. The trade-off between the two cases can be illustrated in the following example: if an individual's human capital is twice of the other's, the ratio of their consumptions in sector 3 will be higher in the case of exogenous than endogenous labor supply. However, on the other hand, the more talented one does not gain any more utilities from having more leisure time than the other in the case of exogenous labor supply. In the event of trade liberalization, the price index in sector 3 of north country experiences the largest decline, and those talented ones constitute the major demand for them. Our results suggest that in the case of exogenous labor supply, the most talented ones in the north country collect the largest gain from trade. This implies that for them the benefit from having proportionally higher consumption in sector 3 overcome the cost of constant leisure choice in the case of exogenous labor supply. In the south country, the price index in sector 1 drops the most, the least talented ones thus benefit the most from it, meanwhile they no longer possess the disadvantage of having less leisure time in the case of exogenous labor supply, and thus they are surely the winner in the case.

In Figure C.3 we plot the utility gap for individuals of the same human capital level from the two countries in the trade economy. The results suggest that only in the benchmark and the "more risk averse" scenario, the least talented ones in the south country enjoy a higher welfare than those from the north. When agents are more risk averse, the utility gap tends to be larger in absolute values than the benchmark results when the utility gap is negative, whereas it is smaller when the utility gap is positive.

There is a sharp difference in the utility gap among the least talented ones between benchmark and the case of CES preference structure. Under the CES preference structure, the utility gap remains positive at all human capital levels, and this implies individuals from the north always exceed those from the south in the utilities. When income elasticity become unitary in the case of CES, the least talented ones do not consume proportionally less than others in sector 3, and north country in turn is the major exporter of goods in sector 3, and this implies the poor ones in the south country consume goods in sector 3 at a higher price than those in the north country. This explains the largest gap appears in the bottom of the human capital distribution. The utility gap also appears to be decreasing with the human capital levels. The sign of the utility gap among those least talented ones also reverses from benchmark to the case of exogenous labor supply. When labor supply is normalized to be 1, the differences in utilities are simply reflected by the differences in consumptions. Those least talented ones in the north country certainly need to pay a higher price than those in the south country for imported goods from sector 1. However, they also get to pay significantly lower price for goods from sector 3 where the north country is the major exporter. Overall, the benefits outweigh the costs and thus they are better off in the north than those in the south country. On the other hand, in the case of endogenous labor supply, individuals from the south can afford the relatively expensive imported goods from sector 3 by proving more labor than those in the north. Overall, they appear to be better off than those from the north. When the labor supply becomes exogenous, the utility gap peaks at human capital slightly below 2.0.

In Figure C.4, we continue to investigate the utility gap in the case of autarky economy. The utility gap remains positive in all the cases except when individuals become more risk averse. As discussed before, individuals tends to reduce labor supply when they become more risk averse, and this leads to a lower income. Since individuals with lower income tend to proportionally spend more on goods produced in sector 1, and price index of sector 1 in the south country is also lower than that in the north country due to a larger market size, and these together explain the negative utility gap at the lower end of the human capital distribution between the two countries. Different from the results in the trade economy, the utility gap monotonically decreases with human capital levels in the case of exogenous labor supply, this is because individuals closer to the bottom of the distribution still have a moderate demand for goods from sector 1 and sector 3, but in the autarky economy they cannot benefit from the lower price index at both sectors compared with the trade economy.

Finally, we compare the changes in the utility gap from autarky to the trade economy in Figure C.5. A negative number implies the utility gap between the countries is shrinking due to trade, otherwise the gap is widening. Similar to the benchmark results, the utility gap is negative at lower end of the distribution and positive at the top of the distribution when individuals become more risk averse. On the other hand, in the case of CES preference structure and exogenous labor supply, the theory predicts convergence in the utility gap between the two countries. In addition, the utility gap shrinks the most at the lowest human capital level, and the changes in the utility gap in absolute values monotonically decrease with the human capital levels. Our results suggest trade liberalization may not necessarily lead to a convergent outcome in welfare between a developed and a developing country. The least talented and thus the poorest one may catch up those in the developed country, but the gap among the richest ones may be amplified. The results depend crucially on the non-homothetic preference structure and endogenous labor supply. Our findings also shed light on policy issues such as how trade protectionism can affect the welfare in both origin and destination countries.

4 Conclusion

In this paper, we develop a trade model with monopolistic competition, non-homothetic preference structure, heterogeneous agents, and endogenous labor supply. Individuals differ in their human capital endowment, and the two countries possess different distribution of human capital. We apply the theoretical framework to quantitatively examine the impacts of trade liberalization at the aggregate, sectoral as well as individual level. The major findings are: trade leads to welfare gain for all the individuals at both countries. In the developing country, the welfare gain tends to be decreasing with human capital level, while in the developed country the welfare gain peaks at a human capital level closer to the bottom of the distribution. Individuals from the developed country always enjoys higher welfare than those in the developing country in the autarky economy, but those least talented ones from the developing country are better off than those in the developed country in the trade economy. The utility gap tends to shrink at the lower end of the distribution, but widens at the higher end of the distribution from autarky to the trade economy. Both non-homothetic preferences and exogenous labor supply are quantitatively important in driving the results.

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Appendix

A Solving the Model

A.1 Individual's optimal consumption and leisure choice

Individual of human capital h chooses the sectoral-level consumption and leisure to solve the following utility maximization problem [4]:

$$\max_{C_s(h),\ell(h)} \frac{C^j\left(h\right)^{1-\gamma}}{1-\gamma}+\theta\frac{\ell\left(h\right)^{1-\gamma}}{1-\gamma},$$
$$s.t \quad \sum p_s C_s\left(h\right)=wh\left(1-\ell\left(h\right)\right).$$

We set-up the Langrange as follows:

$$L=\frac{C\left(h\right)^{1-\gamma}}{1-\gamma}+\theta\frac{\ell\left(h\right)^{1-\gamma}}{1-\gamma}+\lambda\left[wh\left(1-\ell\left(h\right)\right)-\sum p_s C_s\left(h\right)\right].$$

Taking first order conditions with respect to $C_s(h)$ and $\ell(h)$ gives:

$$\begin{aligned} C\left(h\right)^{-\gamma}\frac{\partial C\left(h\right)}{\partial C_s\left(h\right)} &= \lambda p_s, \forall s, \\ \theta\ell\left(h\right)^{-\gamma} &= \lambda wh. \end{aligned}$$

We can further obtain $\frac{\partial C(h)}{\partial C_s(h)}$ from differentiating the recursive utility function

$$\sum_k C\left(h\right)^{\frac{\epsilon(k)-\eta}{\eta}}C_k\left(h\right)^{\frac{\eta-1}{\eta}}=1.$$

Therefore we have:

$$\sum_k \frac{\epsilon\left(k\right)-\eta}{\eta}C\left(h\right)^{\frac{\epsilon(k)-2\eta}{\eta}}\frac{\partial C^j\left(h\right)}{\partial C_s\left(h\right)}C_k\left(h\right)^{\frac{\eta-1}{\eta}}+C\left(h\right)^{\frac{\epsilon(s)-\eta}{\eta}}\frac{\eta-1}{\eta}C_s\left(h\right)^{\frac{-1}{\eta}}=0.$$

[4] To simplify the notation, we skip the country super-script.

That is,

$$\frac{\partial C(h)}{\partial C_s(h)} = \frac{C(h)^{\frac{\epsilon(s)-\eta}{\eta}} \frac{1-\eta}{\eta} C_s(h)^{\frac{-1}{\eta}}}{\sum_k \frac{\epsilon(k)-\eta}{\eta} C(h)^{\frac{\epsilon(k)-2\eta}{\eta}} C_k(h)^{\frac{\eta-1}{\eta}}},$$

$$= \frac{(1-\eta) C(h)^{\frac{\epsilon(s)-\eta}{\eta}} C_s(h)^{\frac{-1}{\eta}}}{\sum_k (\epsilon(k)-\eta) C(h)^{\frac{\epsilon(k)-2\eta}{\eta}} C_k(h)^{\frac{\eta-1}{\eta}}}.$$

The first order condition thus becomes:

$$C(h)^{-\gamma} \frac{(1-\eta) C(h)^{\frac{\epsilon(s)-\eta}{\eta}} C_s(h)^{\frac{-1}{\eta}}}{\sum_k (\epsilon(k)-\eta) C(h)^{\frac{\epsilon(k)-2\eta}{\eta}} C_k(h)^{\frac{\eta-1}{\eta}}} = \frac{\theta \ell(h)^{-\gamma}}{wh} p_s, \forall s. \tag{11}$$

Combining the above with the budget constraint gives:

$$\frac{wh}{\theta \ell(h)^{-\gamma}} \sum_s C(h)^{-\gamma} \frac{(1-\eta) C(h)^{\frac{\epsilon(s)-\eta}{\eta}} C_s(h)^{\frac{\eta-1}{\eta}}}{\sum_k (\epsilon(k)-\eta) C(h)^{\frac{\epsilon(k)-2\eta}{\eta}} C_k(h)^{\frac{\eta-1}{\eta}}} = \sum_s p_s C_s(h) = wh(1-\ell(h)).$$

Rearranging the expression above gives:

$$\sum_s C(h)^{-\gamma} \frac{(1-\eta) C(h)^{\frac{\epsilon(s)-\eta}{\eta}} C_s(h)^{\frac{\eta-1}{\eta}}}{\sum_k (\epsilon(k)-\eta) C(h)^{\frac{\epsilon(k)-2\eta}{\eta}} C_k(h)^{\frac{\eta-1}{\eta}}} = (1-\ell(h)) \theta \ell(h)^{-\gamma}. \tag{12}$$

We multiple wh to both sides of the equation above, and it becomes:

$$\sum_s \frac{(1-\eta) C(h)^{\frac{\epsilon(s)-\eta}{\eta}} C_s(h)^{\frac{\eta-1}{\eta}}}{\sum_k (\epsilon(k)-\eta) C(h)^{\frac{\epsilon(k)-2\eta}{\eta}} C_k(h)^{\frac{\eta-1}{\eta}}} \frac{wh}{\theta} = C(h)^{\gamma} E(h) \ell(h)^{-\gamma},$$

That is,

$$l(h)^{\gamma} = \frac{C(h)^{\gamma} E(h) \theta}{wh} \frac{\sum_k \beta_k^{\frac{1}{\eta}} (\epsilon(k)-\eta) C(h)^{\frac{\epsilon(k)-2\eta}{\eta}} C_k(h)^{\frac{\eta-1}{\eta}}}{\sum_s (1-\eta) \beta_s^{\frac{1}{\eta}} C(h)^{\frac{\epsilon(s)-\eta}{\eta}} C_s(h)^{\frac{\eta-1}{\eta}}}.$$

Given the recursive utility function specification, the above becomes:

$$\begin{aligned} l(h)^{\gamma} &= \frac{C(h)^{\gamma} E(h)\theta}{wh(1-\eta)} \sum_k (\epsilon(k)-\eta) C(h)^{\frac{\epsilon(k)-2\eta}{\eta}} C_k(h)^{\frac{\eta-1}{\eta}}, \\ &= \frac{E(h)\theta}{wh(1-\eta)} \sum_k (\epsilon(k)-\eta) C(h)^{\frac{\epsilon(k)-2\eta}{\eta}+\gamma} C_k(h)^{\frac{\eta-1}{\eta}}. \end{aligned}$$

Moreover, taking the ratio of two arbitrary first order conditions with respect to $C_s(h)$ and $C_m(h)$ gives:

$$C(h)^{\frac{\epsilon(s)-\epsilon(m)}{\eta}} \left(\frac{C_s(h)}{C_m(h)}\right)^{-\frac{1}{\eta}} = \frac{p_s}{p_m}.$$

That is,

$$\frac{C_m(h)}{C_s(h)} = \left[\frac{p_s}{p_m} C(h)^{\frac{\epsilon(m)-\epsilon(s)}{\eta}}\right]^{\eta}.$$

Substituting the above into the budget constraint gives:

$$\sum_t p_t \left[\frac{p_s}{p_t} C(h)^{\frac{\epsilon(t)-\epsilon(s)}{\eta}}\right]^{\eta} C_s(h) = wh(1-\ell(h)).$$

Therefore, the sectoral-level consumption can be obtained as:

$$C_s(h) = \frac{wh(1-\ell(h))}{\sum_t p_t^{1-\eta} p_s^{\eta} C(h)^{\epsilon(t)-\epsilon(s)}} = \frac{E(h) p_s^{-\eta} C_s(h)^{\epsilon(s)-\eta}}{\sum_t p_t^{1-\eta} C(h)^{\epsilon(t)-\eta}}.$$

Substituting the expression for $C_s(h)$ into the recursive utility specification leads to:

$$\sum_s C(h)^{\frac{\varepsilon(s)-\eta}{\eta}} \frac{E^{\frac{\eta-1}{\eta}} \left(C(h)^{\varepsilon(s)-\eta}\right)^{\frac{\eta-1}{\eta}}}{p_s^{\eta-1} \left(\sum_t p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}\right)^{\frac{\eta-1}{\eta}}} = 1.$$

After certain manipulation, we can show the above is equivalent to:

$$E(h)^{1-\eta} \equiv \sum_t p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}. \tag{13}$$

Substituting the expression of $E(h)$ above into the expression of $C_s(h)$ yields:

$$C_s(h) = \frac{E(h)\, C(h)^{\varepsilon(s)-\eta}}{p_s^{\eta} \sum_t p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}} = \frac{E(h)^{\eta}\, C(h)^{\varepsilon(s)-\eta}}{p_s^{\eta}}. \tag{14}$$

Therefore,

$$C_k(h)^{\frac{\eta-1}{\eta}} = \left[\frac{E(h)^{\eta}\, C(h)^{\varepsilon(k)-\eta}}{p_k^{\eta}}\right]^{\frac{\eta-1}{\eta}} = \left(\frac{E(h)}{p_k}\right)^{\eta-1} C(h)^{\frac{(\varepsilon(k)-\eta)(\eta-1)}{\eta}}.$$

Substituting above back into the expression of $l(h)^{\gamma}$, we have

$$\begin{aligned}
l(h)^{\gamma} &= \frac{\theta E(h)}{wh(1-\eta)} \sum_k (\epsilon(k)-\eta)\, C(h)^{\frac{\epsilon(k)-2\eta}{\eta}+\gamma} C_k(h)^{\frac{\eta-1}{\eta}}, \\
&= \frac{\theta E(h)}{wh(1-\eta)} \sum_k (\epsilon(k)-\eta)\, C(h)^{\frac{\epsilon(k)-2\eta}{\eta}+\gamma} \left(\frac{E(h)}{p_k}\right)^{\eta-1} C(h)^{\frac{(\varepsilon(k)-\eta)(\eta-1)}{\eta}}, \\
&= \frac{\theta E(h)^{\eta}}{wh(1-\eta)} \sum_k (\epsilon(k)-\eta)\, p_k^{1-\eta} C^{j}(h)^{(\varepsilon(k)-\eta-1+\gamma)}.
\end{aligned}$$

We further polish the expression above:

$$l(h)^{\gamma} = \frac{\theta E(h)^{\eta-1}(1-\ell(h))}{1-\eta} \sum_k (\epsilon(k)-\eta)\, p_k^{1-\eta} C(h)^{(\varepsilon(k)-\eta-1+\gamma)}. \tag{15}$$

To sum up, given sectoral level prices and wage rate, we can solve $C(h)$ and $\ell(h)$ using Equation (15) and Equation (13). Afterwards, we can obtain sectoral level consumption from Equation (14).

A.2 Consumption and labor supply elasticity

From equation 13, we can have:

$$\frac{\partial \ln C(h)}{\partial \ln E(h)} = \frac{\partial C(h)}{\partial E(h)} \frac{E(h)}{C(h)} = \frac{\sum_t P_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}}{\sum_t \frac{\varepsilon(s)-\eta}{1-\eta} P_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}},$$

and from equation 14 we can obtain:

$$\begin{aligned}\frac{\partial \ln C_s(h)}{\partial \ln E(h)} &= (\epsilon(s)-\eta)\frac{\partial \ln C(h)}{\partial \ln E(h)} + \eta\frac{\partial \ln E(h)}{\partial \ln E(h)} \\ &= (\varepsilon(s)-\eta)\frac{\sum_t P_t^{1-\eta} C(h)^{\epsilon(t)-\eta}}{\sum_t \frac{\epsilon(t)-\eta}{1-\eta} P_t^{1-\eta} C(h)^{\epsilon(t)-\eta}} + \eta\end{aligned}$$

Therefore, the elasticity of human capital against sectoral consumption can be obtained as:

$$\begin{aligned}\frac{\partial \ln C_s(h)}{\partial \ln h} &= \frac{\partial \ln C_s(h)}{\partial \ln E(h)}\frac{\partial \ln E(h)}{\partial \ln h} \\ &= \left\{(\varepsilon(s)-\eta)\frac{\sum_t P_t^{1-\eta} C(h)^{\epsilon(t)-\eta}}{\sum_t \frac{\epsilon(t)-\eta}{1-\eta} P_t^{1-\eta} C(h)^{\epsilon(t)-\eta}} + \eta\right\} * \left\{1 - \frac{\ell(h)}{1-\ell(h)}\frac{\partial \ln \ell(h)}{\partial \ln h}\right\}. \quad (16)\end{aligned}$$

From Equation (15), we have:

$$E(h)^{1-\eta} = \frac{\theta \ell(h)^{-\gamma}(1-\ell(h))}{1-\eta}\sum_k (\epsilon(k)-\eta) p_k^{1-\eta} C(h)^{(\varepsilon(k)-\eta-1+\gamma)}.$$

Together with Equation (13), we can obtain:

$$\sum_t p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta} = \frac{\theta l(h)^{-\gamma}(1-\ell(h))}{1-\eta}\sum_k (\epsilon(k)-\eta) p_k^{1-\eta} C(h)^{(\varepsilon(k)-\eta-1+\gamma)}.$$

Rearraging the expression above gives:

$$\begin{aligned} l(h)^{-\gamma}(1-\ell(h)) &= \left[\frac{\theta}{1-\eta}\frac{\sum_k (\epsilon(k)-\eta) P_k^{1-\eta} C^j(h)^{\varepsilon(k)-1+\gamma}}{\sum_t P_t^{1-\eta} C(h)^{\varepsilon(t)}}\right]^{-1}, \\ &= (\eta-1)\sum_t P_t^{1-\eta} C(h)^{\varepsilon(t)}\left[\theta\sum_k (\eta-\epsilon(k)) P_k^{1-\eta} C^j(h)^{\varepsilon(k)-1+\gamma}\right]^{-1}.\end{aligned}$$

We differentiate above with respective to h, and it becomes:

$$\begin{aligned} & -\gamma\ell(h)^{-\gamma-1}\frac{d\ell(h)}{dh}(1-\ell(h))-\ell(h)^{-\gamma}\frac{d\ell(h)}{dh} \\ = \; & (\eta-1)\left(\sum\varepsilon(t)\,p_t^{1-\eta}C(h)^{\varepsilon(t)-1}\frac{dC(h)}{dh}\right)\left[\theta\sum_k(\eta-\epsilon(k))\,p_k^{1-\eta}C(h)^{\varepsilon(k)-1+\gamma}\right]^{-1} \\ & -(\eta-1)\left(\sum p_t^{1-\eta}C(h)^{\varepsilon(t)}\right)\left[\theta\sum_k(\eta-\epsilon(k))\,p_k^{1-\eta}C(h)^{\varepsilon(k)-1+\gamma}\right]^{-2} \\ & *\left[\theta\sum_k[\eta-\varepsilon(k)]\,\varepsilon(k)\,C(h)^{\varepsilon(k)-1-1+\gamma}\frac{dC(h)}{dh}P_k^{1-\eta}.\right] \end{aligned}$$

Since

$$l(h)^{-\gamma}(1-\ell(h))=\left[\frac{\theta}{1-\eta}\frac{\sum_k(\epsilon(k)-\eta)\,p_k^{1-\eta}C(h)^{\varepsilon(k)-1+\gamma}}{\sum_t p_t^{1-\eta}C(h)^{\varepsilon(t)}}\right]^{-1},$$

so the first part above becomes:

$$\frac{1}{C(h)}\frac{dC(h)}{dh}l(h)^{-\gamma}(1-\ell(h)).$$

Similarly, the second part becomes:

$$\begin{aligned} & (\eta-1)\left(\sum P_t^{1-\eta}C(h)^{\varepsilon(t)}\right)\frac{\theta\sum_k[\eta-\varepsilon(k)]\,\varepsilon(k)\,C(h)^{\varepsilon(k)-1-1+\gamma}\frac{dC(h)}{dh}P_k^{1-\eta}}{\left[\theta\sum_k(\eta-\epsilon(k))\,P_k^{1-\eta}C(h)^{\varepsilon(k)-1+\gamma}\right]^2} \\ = \; & (\eta-1)\left(\sum P_t^{1-\eta}C(h)^{\varepsilon(t)}\right)\left[\theta\sum_k(\eta-\epsilon(k))\,P_k^{1-\eta}C(h)^{\varepsilon(k)-1+\gamma}\right]^{-1}\left[\theta\sum_k(\eta-\epsilon(k))\,P_k^{1-\eta}C(h)^{\varepsilon(k)-1+\gamma}\right]^{-1} * \\ & \left[\theta\sum_k[\eta-\varepsilon(k)]\,\varepsilon(k)\,C(h)^{\varepsilon(k)-2+\gamma}\frac{dC(h)}{dh}P_k^{1-\eta}\right] \\ = \; & l(h)^{-\gamma}(1-\ell(h))\frac{\sum_k[\eta-\varepsilon(k)]\,\varepsilon(k)\,C(h)^{\varepsilon(k)}(P_k)^{1-\eta}}{\sum_k(\eta-\epsilon(k))\,P_k^{1-\eta}C(h)^{\varepsilon(k)}}\frac{1}{C(h)}\frac{dC(h)}{dh}. \end{aligned}$$

Rearranging the results from above, we have:

$$\begin{aligned} & -\gamma\ell(h)^{-\gamma-1}\frac{d\ell(h)}{dh}(1-\ell(h))-\ell(h)^{-\gamma}\frac{d\ell(h)}{dh} \\ = \; & \frac{1}{C(h)}\frac{dC(h)}{dh}l(h)^{-\gamma}(1-\ell(h))-\ell(h)^{-\gamma}(1-\ell(h))\frac{\sum_k[\eta-\varepsilon(k)]\,\varepsilon(k)\,C(h)^{\varepsilon(k)}(P_k)^{1-\eta}}{\sum_k(\eta-\epsilon(k))\,P_k^{1-\eta}C(h)^{\varepsilon(k)}}\frac{1}{C(h)}\frac{dC(h)}{dh}. \end{aligned}$$

That is:

$$\begin{aligned}
& -\frac{d\ell(h)}{dh} h\left[\gamma \ell(h)^{-1}(1-\ell(h))+1\right], \\
= & -\frac{d\ell(h)}{dh} \frac{h}{1-\ell(h)}(1-\ell(h))\left[\gamma \ell(h)^{-1}(1-\ell(h))+1\right], \\
= & \frac{1}{C(h)} \frac{dC(h)}{dh} h(1-\ell(h))\left[1-\frac{\sum_k[\eta-\varepsilon(k)] \varepsilon(k) C(h)^{\varepsilon(k)}\left(P_k\right)^{1-\eta}}{\sum_k(\eta-\epsilon(k)) P_k^{1-\eta} C(h)^{\varepsilon(k)}}\right].
\end{aligned}$$

Let $\xi(\ell)=\frac{d\ell(h)}{dh} \frac{h}{(1-\ell(h))}$, $\xi(c)=\frac{h}{C(h)} \frac{dC(h)}{dh}$, so they in turn correspond to the labor supply elasticity and consumption elasticity, respectively. The above can be rewritten as:

$$-\xi(\ell)\left[\gamma \ell(h)^{-1}+(1-\gamma)\right]=\xi(c)\left[1-\frac{\sum_k[\eta-\varepsilon(k)] \varepsilon(k) C(h)^{\varepsilon(k)} P_k^{1-\eta}}{\sum_k(\eta-\epsilon(k)) P_k^{1-\eta} C(h)^{\varepsilon(k)}}\right] \tag{17}$$

On the other hand, since

$$\begin{aligned}
E(h)^{1-\eta} & = \sum_{t=1}^S p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}, \\
wh(1-\ell(h)) & = \left(\sum_{t=1}^S p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}\right)^{\frac{1}{1-\eta}}, \\
\ell(h) & = 1-\frac{\left(\sum_{t=1}^S p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}\right)^{\frac{1}{1-\eta}}}{wh},
\end{aligned}$$

and thus we have:

$$\begin{aligned}
\frac{d\ell(h)}{dh} = & -\frac{\frac{1}{1-\eta}\left(\sum_{t=1}^S p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}\right)^{\frac{\eta}{1-\eta}}}{wh}\left(\sum_{t=1}^S(\varepsilon(t)-\eta) p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta-1} \frac{dC(h)}{dh}\right) \\
& +\frac{\left(\sum_{t=1}^S p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}\right)^{\frac{1}{1-\eta}}}{wh^2}.
\end{aligned}$$

Rearraging the expression above, we have:

$$
\begin{aligned}
\frac{d\ell(h)}{dh} wh^2 - E(h) &= \frac{h}{\eta-1} E(h)^{\eta} \left(\sum_{t=1}^{S} (\varepsilon(t) - \eta) p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta-1} \right) \frac{dC(h)}{dh}, \\
\frac{d\ell(h)}{dh} wh - w(1-\ell(h)) &= \frac{E(h)^{\eta}}{\eta-1} \left(\sum_{t=1}^{S} (\varepsilon(t) - \eta) p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta-1} \right) \frac{dC(h)}{dh}, \\
\frac{d\ell(h)}{dh} \frac{wh(1-\ell(h))}{(1-\ell(h))} - \frac{wh(1-\ell(h))}{h} &= \frac{E(h)^{\eta}}{\eta-1} \left(\sum_{t=1}^{S} (\varepsilon(t) - \eta) p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta-1} \right) \frac{dC(h)}{dh}, \\
\frac{d\ell(h)}{dh} \frac{E(h)}{(1-\ell(h))} - \frac{E(h)}{h} &= \frac{E(h)^{\eta}}{\eta-1} \left(\sum_{t=1}^{S} (\varepsilon(t) - \eta) p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta-1} \right) \frac{dC(h)}{dh}, \\
\frac{d\ell(h)}{dh} \frac{1}{(1-\ell(h))} - \frac{1}{h} &= \frac{E(h)^{\eta-1}}{\eta-1} \left(\sum_{t=1}^{S} (\varepsilon(t) - \eta) p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta-1} \right) \frac{dC(h)}{dh}, \\
\frac{d\ell(h)}{dh} \frac{h}{(1-\ell(h))} - 1 &= \frac{1}{\eta-1} \frac{\left(\sum_{t=1}^{S} (\varepsilon(t) - \eta) p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta} \right)}{\sum_t p_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}} \frac{dC(h)}{dh} \frac{h}{C(h)}.
\end{aligned}
$$

Finally, the above is equivalent to:

$$
\xi(\ell) - 1 = \frac{1}{\eta-1} \frac{\left(\sum_{t=1}^{S} (\varepsilon(t) - \eta) P_t^{1-\eta} C(h)^{\varepsilon(t)-\eta} \right)}{\sum_t P_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}} \xi(c). \tag{18}
$$

To sum up, given prices and the optimal consumption and leisure choice, the human capital elasticity of labor supply and final consumption at each human capital level h can be solved from Equation (17) and Equation (18). In addition, the sectoral level consumption elasticity can be obtained from Equation (16).

In the special case of CES preference, we can explicitly solve for a constant level of $\xi(\ell)$ and $\xi(c)$ as:

$$
\begin{aligned}
\xi(\ell) &= \frac{(\eta-1)(\varepsilon-1)}{(\eta-1)(\varepsilon-1) + (\eta-\varepsilon)(1-\gamma+\gamma\frac{1}{\ell}}, \\
\xi(c) &= \frac{(1-\gamma+\gamma\frac{1}{\ell})(\eta-1)}{(\eta-\varepsilon)(1-\gamma+\frac{\gamma}{\ell}) + (\eta-1)(\varepsilon-1)}.
\end{aligned}
$$

Both elasticities can be found to be independent on the human capital level h.

A.3 Solving for Net Exports

In a two-country setting, in this section, we intend to solve each country's net export. Since

$$D_s^{ij}(v) = N^j \frac{\int_{\underline{h}}^{\overline{h}} E^j(h)^\eta C^j(h)^{\varepsilon(s)-\eta} dG(h)}{P_s^{j\eta-\sigma}} * p_s^{ij}(v)^{-\sigma}.$$

We define b_s^j as follows:

$$b_s^j = N^j \frac{\int_{\underline{h}}^{\overline{h}} E^j(h)^\eta C^j(h)^{\varepsilon(s)-\eta} dG(h)}{P_s^{j\eta-\sigma}}.$$

From the expression of expenditure share 9, we can have:

$$b_s^j = m_s^j P_s^{j\sigma-1} w^j L_j^e. \tag{19}$$

We also denote f_s^j to be the fraction of firms in sector s in country j:

$$f_s^j = \frac{V_s^j}{\sum_t V_t^j}.$$

The net export in country i of sector s specified in equation 7 can be expressed as:

$$NX_s^i = f_s^i b_s^j \rho \left(w^i \psi_s^i \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \sum_t V_t^i - f_s^j b_s^i \rho \left(w^j \psi_s^j \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \sum_t V_t^j. \tag{20}$$

Zero profit condition implies the quantity of goods produced by each firm in country i sector s should equals to $\frac{(\sigma-1)\phi_s^i}{\psi_s^i}$, so

$$\sum_j \tau_{ij} D_s^{ij}(v) = \sum_j \tau_{ij} b_s^j p_s^{ij}(v)^{-\sigma} = \frac{(\sigma - 1)\phi_s^i}{\psi_s^i},$$

then we can obtain the expression for b_s^i as follows:

$$b_s^1 = \frac{\frac{(\sigma-1)\phi_s^1}{\psi_s^1}\left(w^1\psi_s^1\frac{\sigma}{\sigma-1}\right)^\sigma - \frac{(\sigma-1)\phi_s^2}{\psi_s^2}\left(w^2\psi_s^2\frac{\sigma}{\sigma-1}\right)^\sigma\rho}{1-\rho^2}, \tag{21}$$

$$b_s^2 = \frac{\frac{(\sigma-1)\phi_s^2}{\psi_s^2}\left(w^2\psi_s^2\frac{\sigma}{\sigma-1}\right)^\sigma - \frac{(\sigma-1)\phi_s^1}{\psi_s^1}\left(w^1\psi_s^1\frac{\sigma}{\sigma-1}\right)^\sigma\rho}{1-\rho^2}. \tag{22}$$

Given the definiton of f_s^i, the number of firms in sector s of country i equals $f_s^i \sum_t V_t^i$. By equating equation (19) and (21)-(22), we can get:

$$P_{1s}^{1-\sigma} = \frac{\left(1-\rho^2\right) m_s^1 w^1 L_1^e}{\frac{(\sigma-1)\phi_s^1}{\psi_s^1}\left(w^1\psi_s^1\frac{\sigma}{\sigma-1}\right)^\sigma - \frac{(\sigma-1)\phi_s^2}{\psi_s^2}\left(w^2\psi_s^2\frac{\sigma}{\sigma-1}\right)^\sigma\rho},$$

$$P_{2s}^{1-\sigma} = \frac{\left(1-\rho^2\right) m_s^2 w^2 L_2^e}{\frac{(\sigma-1)\phi_s^2}{\psi_s^2}\left(w^2\psi_s^2\frac{\sigma}{\sigma-1}\right)^\sigma - \frac{(\sigma-1)\phi_s^1}{\psi_s^1}\left(w^1\psi_s^1\frac{\sigma}{\sigma-1}\right)^\sigma\rho}.$$

In addition, the sector level price can also be expressed in the following way:

$$P_{1s}^{1-\sigma} = p_s^{11}(v)^{1-\sigma} f_s^1 \sum_t V_t^1 + p_s^{21}(v)^{1-\sigma} f_s^2 \sum_t V_t^2$$

$$P_{2s}^{1-\sigma} = p_s^{12}(v)^{1-\sigma} f_s^1 \sum_t V_t^1 + p_s^{22}(v)^{1-\sigma} f_s^2 \sum_t V_t^2$$

Equating the above two different expressions of sector level prices gives:

$$f_s^1 = \frac{\frac{m_s^1 w^1 L_1^e}{\frac{\phi_s^1}{\psi_s^1}\left(\psi_s^1\right)^\sigma (w^1)^\sigma - \frac{\phi_s^2}{\psi_s^2}\left(\psi_s^2\right)^\sigma \rho (w^2)^\sigma} - \frac{\rho m_s^2 w^2 L_2^e}{\frac{\phi_s^2}{\psi_s^2}\left(\psi_s^2\right)^\sigma (w^2)^\sigma - \frac{\phi_s^1}{\psi_s^1}\left(\psi_s^1\right)^\sigma \rho (w^1)^\sigma}}{\sigma\left(\psi_s^1\right)^{1-\sigma}(w^1)^{1-\sigma}\sum_t V_t^1}, \tag{23}$$

$$f_s^2 = \frac{\frac{m_s^2 w^2 L_2^e}{\frac{\phi_s^2}{\psi_s^2}\left(\psi_s^2\right)^\sigma (w^2)^\sigma - \frac{\phi_s^1}{\psi_s^1}\left(\psi_s^1\right)^\sigma \rho (w^1)^\sigma} - \frac{\rho m_s^1 w^1 L_1^e}{\frac{\phi_s^1}{\psi_s^1}\left(\psi_s^1\right)^\sigma (w^1)^\sigma - \frac{\phi_s^2}{\psi_s^2}\left(\psi_s^2\right)^\sigma \rho (w^2)^\sigma}}{\sigma\left(\psi_s^2\right)^{1-\sigma}(w^2)^{1-\sigma}\sum_t V_t^2}. \tag{24}$$

By substituting the expression of f_s^j from equation (23)-(24), and the expression of b_s^j from equation (21)-(22) into the expression of net export in equation 20, we can finally get the expression of net export in country i as:

$$NX_s^i = \rho L_j^e w^j \left\{ \frac{\frac{\phi_s^j}{\psi_s^j}\left(\psi_s^j\right)^\sigma m_s^i \omega \frac{L_i^e}{L_j^e}}{\frac{\phi_s^i}{\psi_s^i}\left(\psi_s^i\right)^\sigma \omega^\sigma - \frac{\phi_s^j}{\psi_s^j}\left(\psi_s^j\right)^\sigma \rho} - \frac{\frac{\phi_s^i}{\psi_s^i}\left(\psi_s^i\right)^\sigma \omega^\sigma m_s^j}{\frac{\phi_s^j}{\psi_s^j}\left(\psi_s^j\right)^\sigma - \frac{\phi_s^i}{\psi_s^i}\left(\psi_s^i\right)^\sigma \rho \omega^\sigma} \right\}.$$

Since the two countries are only different in the human capital distribution, so we can simplify the above net export expression as the following:

$$NX_s^i = \rho L_j^e w^j \left\{ \frac{m_s^i \omega}{\omega^\sigma - \rho} \frac{L_i^e}{L_j^e} - \frac{\omega^\sigma m_s^j}{1 - \rho \omega^\sigma} \right\}.$$

B Proofs

Proof for Proposition 1

Proof: (i)(ii) Human capital elasticity of labor supply and final goods consumption can be obtained from Equation ((17)) and ((18). We can observe a linear relation between $\xi(\ell)$ and $\xi(c)$ from both equations. When $\varepsilon(s) > 1$ in all the sectors, it is straightforward to show:

$$\sum_k \left[\eta - \varepsilon(k)\right] \varepsilon(k) C(h)^{\varepsilon(k)} P_k^{1-\eta} > \sum_k \left[\eta - \varepsilon(k)\right] C(h)^{\varepsilon(k)} P_k^{1-\eta}.$$

Moreover, when $\gamma \geq 1$, we also have:

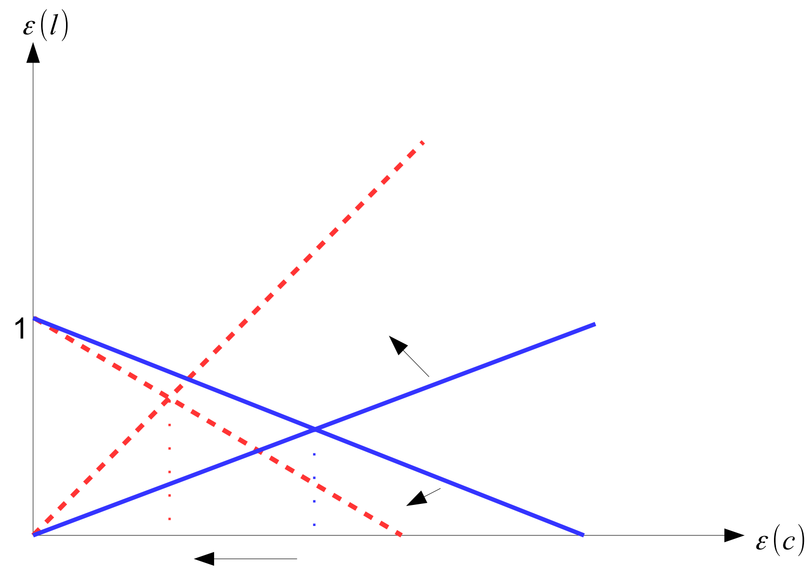
$$\gamma \ell(h)^{-1} + (1-\gamma) = \gamma \frac{1-\ell}{\ell} + 1 > 0.$$

Therefore, we can draw a upward slopping straight line between $\xi(\ell)$ and $\xi(c)$ from Equation (17).

It is also straightforward to show that when $\varepsilon(s) < \eta, \forall s$, there exhibits a negative relation between $\xi(\ell)$ and $\xi(c)$. In addition, the intercept on the vertical axis equals to 1 if we draw $\xi(c)$ at the horizontal axis against $\xi(\ell)$ on the vertical axis.

The analysis above suggest a unique solution of $(\xi(c), \xi(\ell))$ from Equation ((17)) and ((18). Moreover, $0 < \xi(\ell) < 1$ and $\xi(c) > 0$ hold in the solution.

(iii) We can show both $\frac{\sum_k [\eta - \varepsilon(k)] \varepsilon(k) C(h)^{\varepsilon(k)} P_k^{1-\eta}}{\sum_k (\eta - \epsilon(k)) P_k^{1-\eta} C(h)^{\varepsilon(k)}}$ and $\frac{\sum_{t=1}^S (\varepsilon(t) - \eta) P_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}}{\sum_t P_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}}$ are increasing with $c(h)$. Together with the fact that $c(h)$ is increasing in h, as illustrated in the following diagram, a higher human capital level push both blue curves towards the direction of red curves, and thus it results in a lower level of $\xi(c)$ as human capital gets larger. ∎



C Tables and Figures

Table C.1: Benchmark Parameterizations

Para.	**Sources/Targets**	**Para. Value**
ψ_{11}	employment share in agriculture sector of south	0.831
ψ_{21}	normalization	1
ψ_{31}	employment share in service sector of south	1.131
ψ_{12}	employment share in agriculture sector of north	0.766
ψ_{22}	employment share in manufacturing sector of north	1
ψ_{32}	employment share in service sector of north	1.077
ϕ_{11}	fraction of firms in agriculture sector of south	0.663
ϕ_{21}	fraction of entry cost in sales revenue in south	0.711
ϕ_{31}	fraction of firm in service sector of south	0.736
ϕ_{12}	fraction of firm in agricultural sector of north	1.718
ϕ_{22}	fraction of entry cost in sales revenue in north	0.756
ϕ_{32}	fraction of firm in service sector of north	0.969
ϵ_1	wage elasticity in agriculture sector	0.5
ϵ_2	wage elasticity in manufacture sector	1.25
ϵ_3	income elasticity in service sector	1.90
α	income 90/10 ratio	2.02
θ	share of leisure time	0.16
σ	elasticity of substitution within sectors	6
η	price elasticity	4
γ	risk aversion coefficient	2

Note: We calibrate the sectoral labor requirement and entry fee to mimic an even distribution of employment and the number of firms across sectors in a north-south economy. The elasticity of substitution is chosen to match the income elasticity of agricultural, manufacturing, and service goods estimated in the literature. The shape parameter in the less talented country is calibrated to mimic a 90-10 income ratio at 3.2. The iceberg trade is chosen to match an average 30 percent of trade share in both countries. Other parameters we choose either by normalization or by following the literature. For more details, see the main text.

Table C.2: Benchmark Results: aggregate

	GDP	Welfare	Trade Share	Gini_Inc.	9010 Inc.R.
			Trade		
South	1.221	-0.725	0.201	0.216	3.312
North	1.484	-0.675	0.167	0.228	3.698
			Autarky		
South	1.236	-0.739	0.000	0.216	3.319
North	1.489	-0.685	0.000	0.228	3.714

Note: We evaluate the impact of trade liberalization on the aggregate outcomes in the benchmark model with non-homothetic preference structure, endogenous labor supply and the risk aversion coefficient is set to be 2. For more details, see the main text.

Table C.3: Benchmark Results: sector

	South			North		
	Sector 1	Sector 2	Sector 3	Sector 1	Sector 2	Sector 3
			Trade			
Employment	0.499	0.301	0.199	0.199	0.301	0.499
Expenditure	0.390	0.304	0.307	0.268	0.301	0.431
Firm Dist	0.500	0.299	0.200	0.100	0.399	0.500
Net Export	12.057	0.070	-12.007	-12.057	-0.070	12.007
Prices	0.724	0.896	0.996	0.784	0.889	0.911
			Autarky			
Employment	0.397	0.304	0.299	0.293	0.300	0.408
Expenditure	0.380	0.306	0.314	0.280	0.303	0.418
Firm Dist	0.394	0.303	0.304	0.155	0.419	0.425
Net Export	0.000	-0.000	-0.000	-0.000	0.000	0.000
Prices	0.754	0.921	1.015	0.789	0.904	0.937

Note: We evaluate the impact of trade liberalization on the sectoral outcomes in the benchmark model with non-homothetic preference structure, endogenous labor supply and the risk aversion coefficient is set to be 2. For more details, see the main text.

Table C.4: CES Preference: aggregate

	GDP	Welfare	Trade Share	Gini_Inc.	9010 Inc.R.
			Trade		
South	1.233	-0.720	0.194	0.216	3.308
North	1.479	-0.673	0.162	0.231	3.773
			Autarky		
South	1.233	-0.734	0.000	0.216	3.317
North	1.485	-0.683	0.000	0.231	3.783

Note: We evaluate the impact of trade liberalization on the aggregate outcomes in the model with homothetic CES preference structure. We let ε be the simple average of ε_s over the three sectors in the benchmark specification. The remaining parameters remain the same as the benchmark values. For more details, see the main text.

Table C.5: CES Preference: sector

		South			North	
	Sector 1	Sector 2	Sector 3	Sector 1	Sector 2	Sector 3
			Trade			
Employment	0.678	0.228	0.094	0.463	0.292	0.245
Expenditure	0.585	0.256	0.159	0.515	0.277	0.208
Firm Dist	0.680	0.226	0.095	0.268	0.447	0.284
Net Export	10.227	-2.998	-7.324	-10.227	2.998	7.324
Prices	0.702	0.925	1.085	0.728	0.894	0.985
			Autarky			
Employment	0.589	0.255	0.156	0.542	0.268	0.190
Expenditure	0.573	0.261	0.166	0.526	0.275	0.199
Firm Dist	0.586	0.255	0.159	0.334	0.435	0.231
Net Export	0.000	-0.000	-0.000	-0.000	0.000	0.000
Prices	0.721	0.937	1.090	0.736	0.914	1.018

Note: We evaluate the impact of trade liberalization on the sectoral outcomes in the model with homothetic CES preference structure. We let ε be the simple average of ε_s over the three sectors in the benchmark specification. The remaining para-maters remain the same as the benchmark values. For more details, see the main text.

Table C.6: Exogenous Labor: aggregate

	GDP	Welfare	Trade Share	Gini_Inc.	9010 Inc.R.
			Trade		
South	1.858	-0.361	0.214	0.276	4.902
North	2.516	-0.359	0.158	0.307	6.587
			Autarky		
South	1.893	-0.373	0.000	0.276	4.902
North	2.516	-0.366	0.000	0.307	6.587

Note: We evaluate the impact of trade liberalization on the aggregate outcomes in the model with exogenous labor supply. We let θ be zero. The remaining para-maters remain the same as the benchmark values. For more details, see the main text.

Table C.7: Exogenous Labor: sector

	South			North		
	Sector 1	Sector 2	Sector 3	Sector 1	Sector 2	Sector 3
				Trade		
Employment	0.386	0.327	0.287	0.094	0.253	0.653
Expenditure	0.281	0.305	0.414	0.157	0.266	0.577
Firm Dist	0.387	0.324	0.288	0.046	0.323	0.631
Net Export	17.235	4.083	-21.133	-17.235	-4.083	21.133
Prices	0.707	0.844	0.909	0.787	0.853	0.836
				Autarky		
Employment	0.287	0.307	0.406	0.174	0.269	0.557
Expenditure	0.273	0.306	0.422	0.165	0.270	0.565
Firm Dist	0.284	0.305	0.411	0.088	0.359	0.553
Net Export	0.000	-0.000	-0.000	-0.000	0.000	0.000
Prices	0.748	0.881	0.939	0.791	0.865	0.857

Note: We evaluate the impact of trade liberalization on the sectoral outcomes in the model with exogenous labor supply. We let θ be zero. The remaining para-maters remain the same as the benchmark values. For more details, see the main text.

Table C.8: Risk Aversion: aggregate

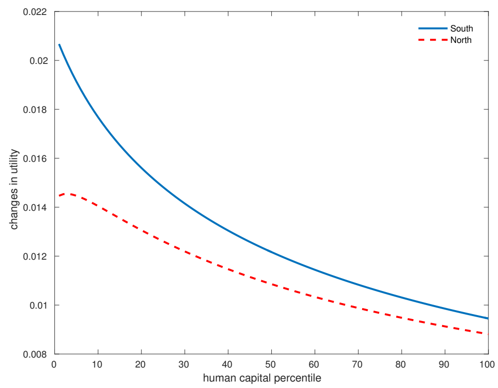
	GDP	Welfare	Trade Share	Gini_Inc.	9010 Inc.R.
			Trade		
South	0.947	-0.421	0.192	0.163	2.395
North	1.080	-0.386	0.172	0.164	2.438
			Autarky		
South	0.955	-0.433	0.000	0.163	2.402
North	1.088	-0.395	0.000	0.165	2.451

Note: We evaluate the impact of trade liberalization on the aggregate outcomes in the model where agents are more risk averse than the benchmark economy. We let γ be 3. The remaining para-maters remain the same as the benchmark values. For more details, see the main text.

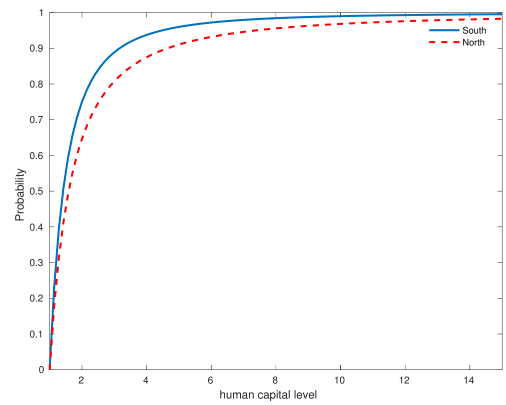
Table C.9: Risk Aversion: sector

	South			North		
	Sector 1	Sector 2	Sector 3	Sector 1	Sector 2	Sector 3
			Trade			
Employment	0.574	0.275	0.151	0.295	0.313	0.392
Expenditure	0.469	0.290	0.240	0.362	0.304	0.334
Firm Dist	0.575	0.273	0.152	0.154	0.434	0.411
Net Export	8.833	-1.090	-7.656	-8.833	1.090	7.656
Prices	0.738	0.936	1.063	0.786	0.919	0.971
			Autarky			
Employment	0.476	0.290	0.234	0.390	0.299	0.311
Expenditure	0.459	0.293	0.247	0.375	0.304	0.321
Firm Dist	0.473	0.289	0.238	0.218	0.441	0.341
Net Export	0.000	-0.000	-0.000	-0.000	0.000	0.000
Prices	0.761	0.953	1.074	0.792	0.937	1.002

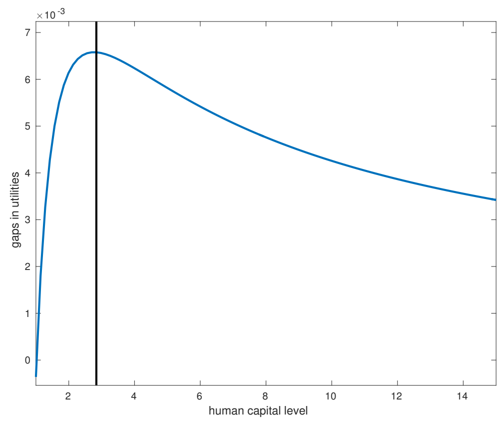
Note: We evaluate the impact of trade liberalization on the sectoral outcomes in the model where agents are more risk averse than the benchmark economy. We let γ be 3. The remaining para-maters remain the same as the benchmark values. For more details, see the main text.



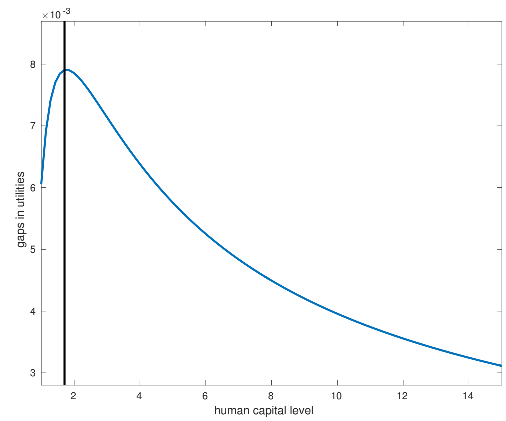
(a) Gains from Trade



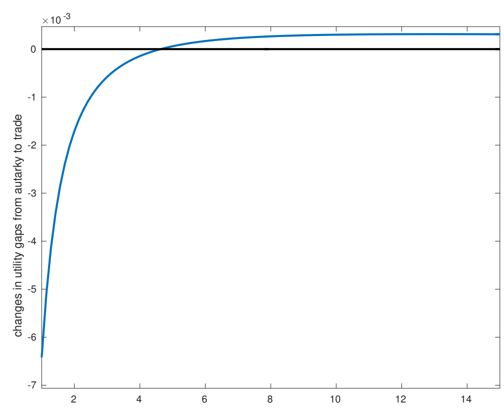
(b) Probability Distribution



(c) Utility Gap in Trade



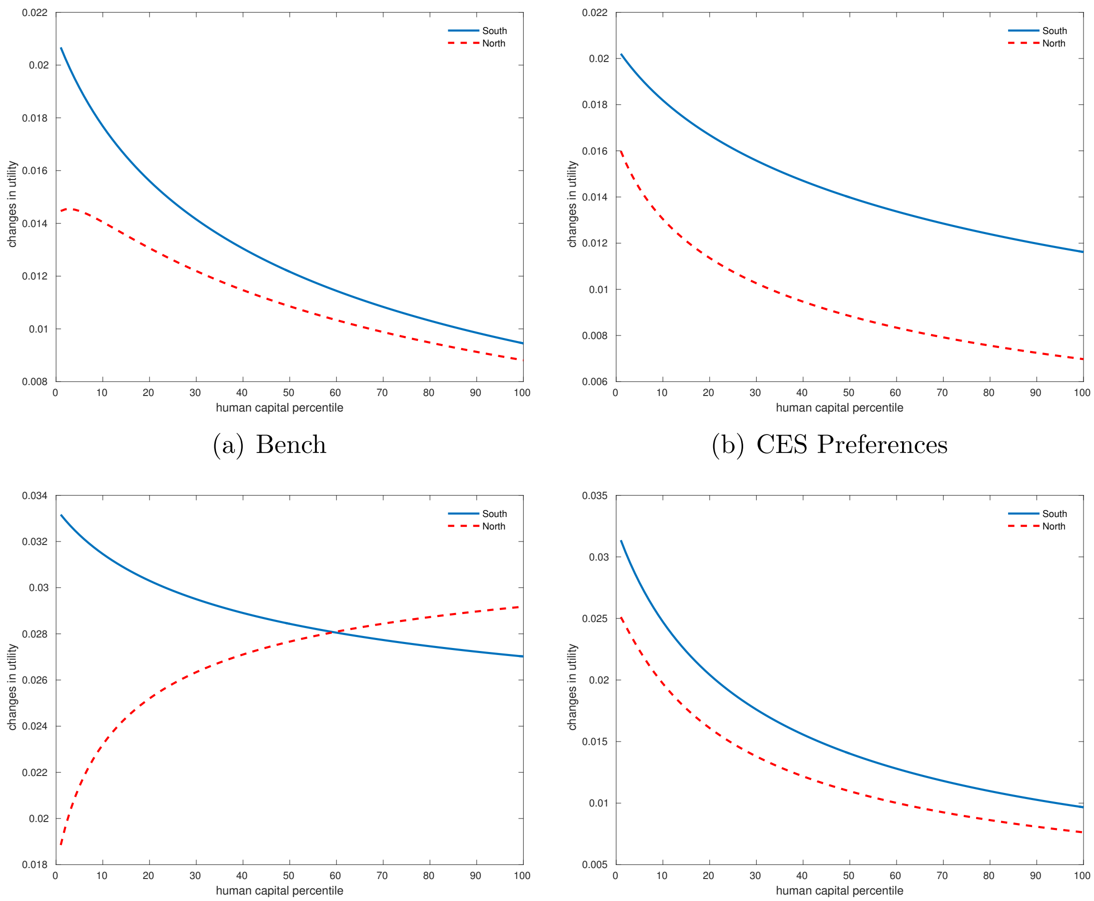
(d) Utility Gap in Autarky



(e) Changes in the Utility Gap

Figure C.1: Individual Results

Notes: Panel (a) plots the percentage change in utilities from autarky to the trade economy against human capital percentile in both countries. Panel (b) plots the corresponding probabilities for a given human capital level in both countries. Panel (c) plots the utility gap between north and south country against each specific human capital in the trade economy. A positive number implies individual of the same human capital level gains higher utility in the north than the south country. Panel (d) plots the utility gap between north and south country against each specific human capital in the autarky economy. Panel (e) plots the changes in the utility gap between the two countries from autarky to trade economy against each specific human capital level. A positive number refers to a widening gap, and a negative number implies a shrinking gap.



(a) Bench

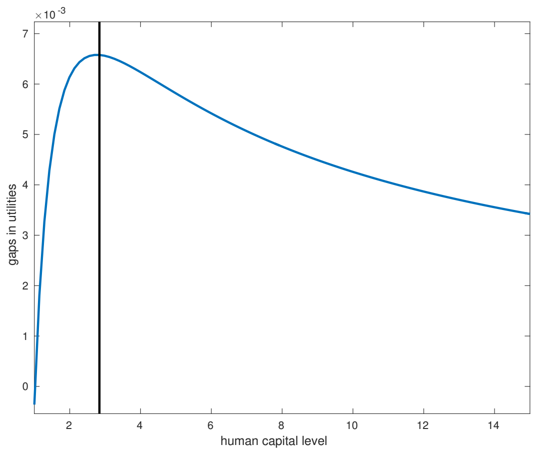
(b) CES Preferences

(c) Exogenous Labor

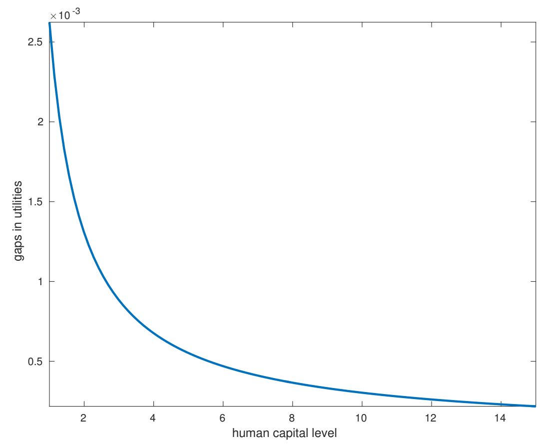
(d) More Risk Averse

Figure C.2: Individual Results: Utility Comparison

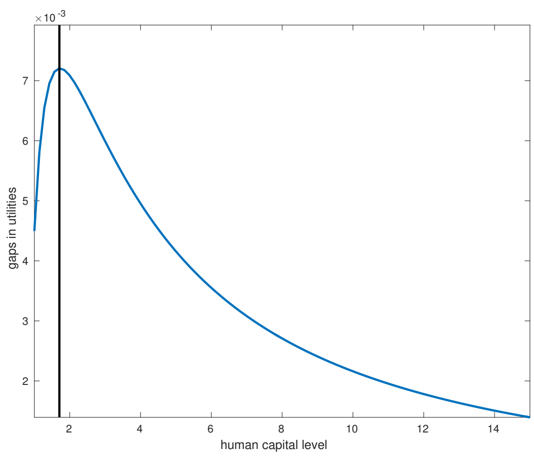
Notes: This figure plots the utility gap between north and south country against each specific human capital in the trade economy. A positive number implies individual of the same human capital level gains higher utility in the north than the south country. Panel (a) is the benchmark economy. Panel (b) has the CES homothetic preference structure. Labor supply is exogenous in Panel (c). Individuals are more risk averse in Panel (d) with a risk averse coefficient at 3.



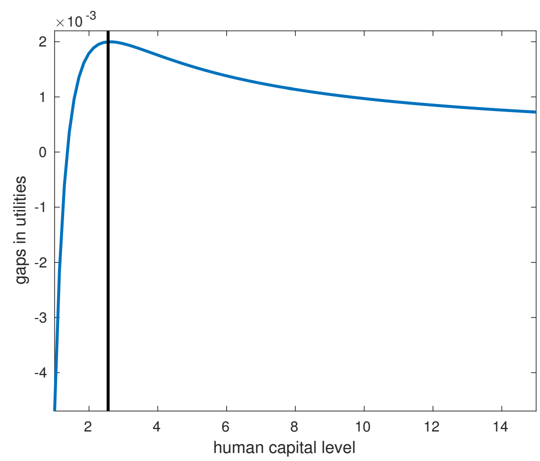
(a) Benchmark



(b) CES Preferences



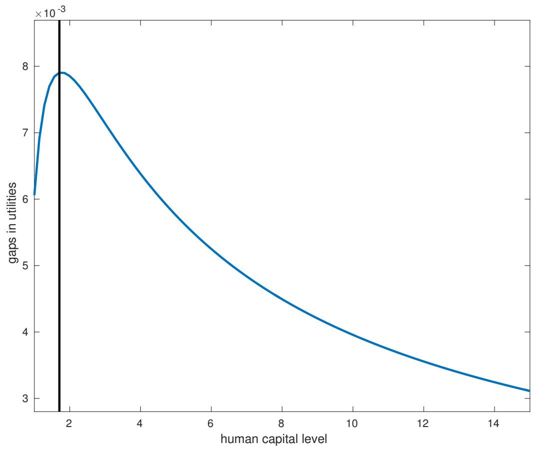
(c) Exogenous Labor



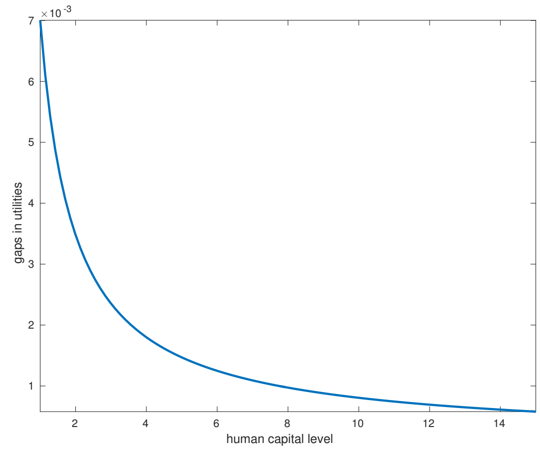
(d) More Risk Averse

Figure C.3: Individual Results: Utility Gap in the Trade Economy

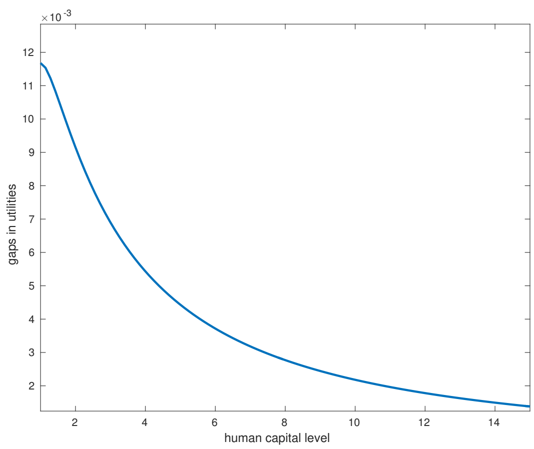
Notes: This figure plots the utility gap between north and south country against each specific human capital in the trade economy. A positive number implies individual of the same human capital level gains higher utility in the north than the south country. Panel (a) is the benchmark economy. Panel (b) has the CES homothetic preference structure. Labor supply is exogenous in Panel (c). Individuals are more risk averse in Panel (d) with a risk averse coefficient at 3.



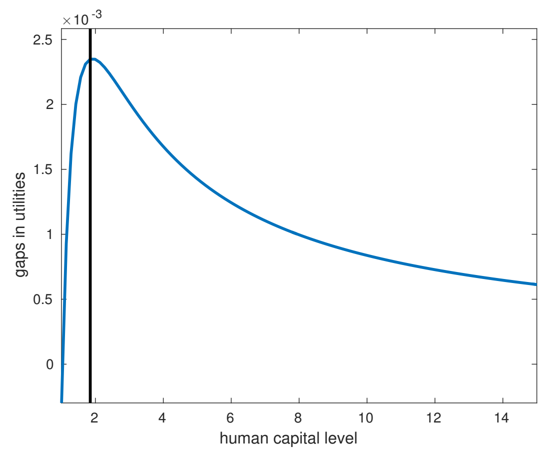
(a) Benchmark



(b) CES Preferences



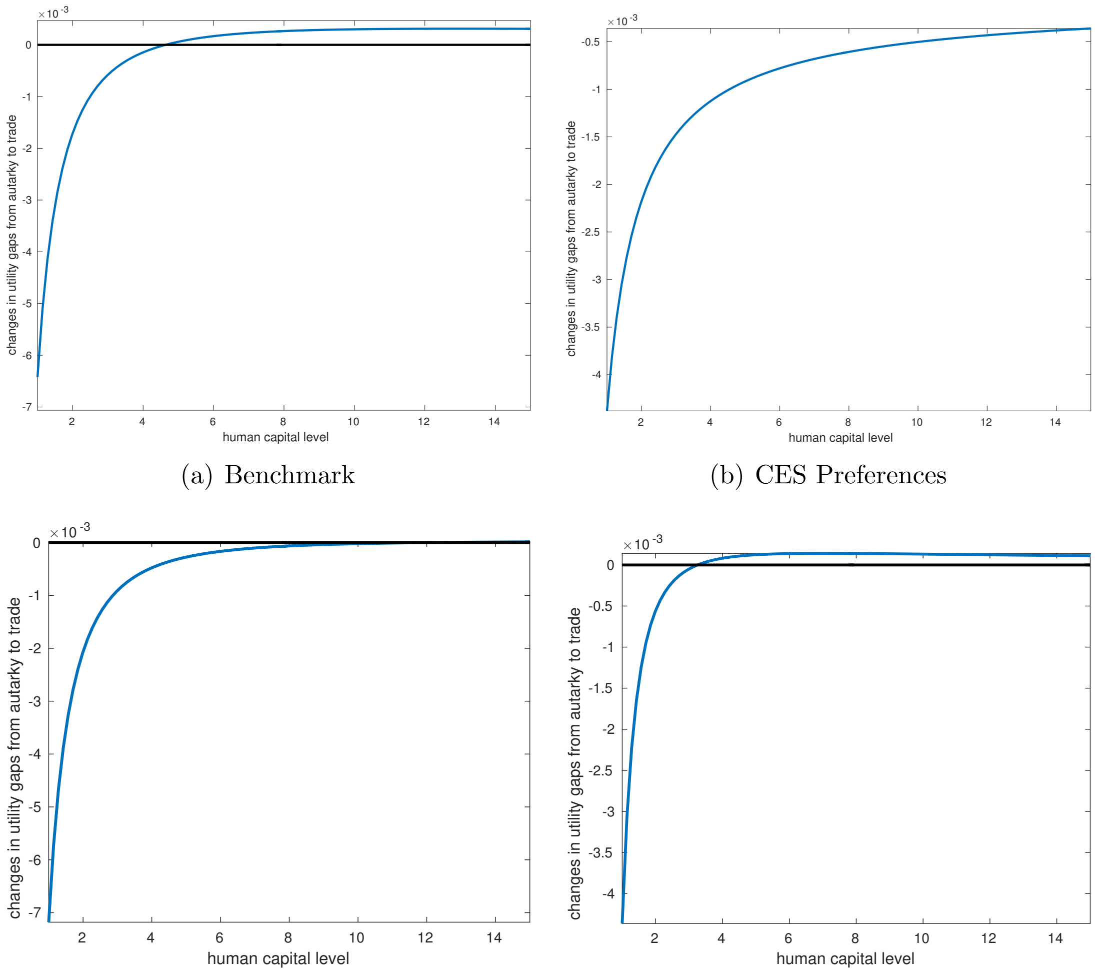
(c) Exogenous Labor



(d) More Risk Averse

Figure C.4: Individual Results: Utility Gap in the Autarky Economy

Notes: This figure plots the utility gap between north and south country against each specific human capital in the autarky economy. A positive number implies individual of the same human capital level gains higher utility in the north than the south country. Panel (a) is the benchmark economy. Panel (b) has the CES homothetic preference structure. Labor supply is exogenous in Panel (c). Individuals are more risk averse in Panel (d) with a risk averse coefficient at 3.



(a) Benchmark

(b) CES Preferences

(c) Exogenous Labor

(d) More Risk Averse

Figure C.5: Individual Results: Changes in Utility Gap from Autarky to Trade Economy

Notes: This figure plots the changes in the utility gap between the two countries from autarky to trade economy against each specific human capital level. A positive number refers to a widening gap, and a negative number implies a shrinking gap. Panel (a) is the benchmark economy. Panel (b) has the CES homothetic preference structure. Labor supply is exogenous in Panel (c). Individuals are more risk averse in Panel (d) with a risk averse coefficient at 3.