Non-homothetic Preferences, Pattern of Trade and Inequality

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Abstract

This paper explores the distributional impacts of non-homothetic CES preference structure and trade liberalization. We embed non-homothetic preferences into a trade model with monopolistic competition. Individuals are heterogeneous in their human capital endowment, and countries only differ in the distribution of human capital. By introducing elastic labor supply, sectoral consumption demand becomes less responsive to changes in human capital than in the case of exogenous labor supply. We parameterize the model to mimic certain stylized facts in the real-world counterpart. The quantitative results suggest that the more talented country enjoys higher GDP and average living standards. However, individuals at the bottom percentile may be better off in the less talented country. The resource distribution across sectors becomes identical in both countries when non-homothetic preferences are removed. Trade liberalization tends to reduce inequality both within and between countries.

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1 Introduction

A growing body of recent literature has aimed to embed non-homothetic CES preferences into the class of growth and new trade models. [1] Examples include Matsuyama [2015] and Comin, Lashkari, and Mestieri [2015]. These studies focus on examining how changes in preference structure affect the aggregate performance of the economy and the pattern of trade across countries. However, little attention has been devoted to how individuals of different types may be influenced. These questions are potentially important if we wish to understand the role of trade liberalization in inequality. This paper attempts to address these questions.

In this paper, we develop a trade model featuring individuals who are heterogeneous in their human capital endowment. Countries are all identical except in the distribution of human capital. Non-homothetic preference structure is introduced together with endogenous labor supply decision. Essentially, an individual's utility level positively depends on final consumption and leisure time, while final consumption is an implicit non-homothetic CES aggregator of sectoral-level consumption. At a lower tier, sectoral consumption is a homothetic CES aggregator of all varieties available in the sector. The production side of the economy is modeled as monopolistic competition. Each firm is the single producer of one variety. Free entry prevails in each sector.

The theoretical framework has the following distinctive features. First, by introducing a tradeoff between labor supply and leisure, income is no longer exogenous. Therefore, instead of conventionally defining an income elasticity, we focus on the elasticity of human capital against sectoral consumption. We show that human capital elasticity can be decomposed into wage elasticity and elasticity of human capital against labor supply, and the latter is proven to be less than one with reasonable parameterization. Therefore, sectoral consumption is less responsive to changes in human capital than in wages. Second, the non-homothetic preference structure enables us to rank the elasticity of human capital against sectoral consumption over sectors. Therefore, individuals with higher human capital tends to proportionally spend more on the sector with higher human capital elasticity. Third,

[1] The New trade theory is proposed in Krugman [1980], and developed by Melitz [2003] and Eaton and Kortum [2002]. They usually feature homothetic preferences and monopolistic or perfect competition.

in the case of open economy, we model the two countries as identical ones except for the initial distribution of human capital. The more talented country thus demands more from the sector with the highest human capital elasticity. Due to the home market effect, more varieties will be produced and the sectoral price will tend to be lower, and thus the more talented country is more likely to become the net exporter of the sectoral varieties.

We calibrate the model into a closed economy with even resource distribution across the sectors. In the case of open economy, we let the human capital distribution in the talented country first order stochastically dominate that in the other country. We compare selected individuals at the same percentile from the human capital distribution in each country. The benchmark results suggest individuals at the bottom percentile in the less talented country tend to obtain higher utilities than those from the talented country. The main reason is that the price level for the sector with least human capital elasticity is lower in less talented country, as it contains a larger market demand. Individuals at the bottom percentile spend proportionally more on the sector with lowest human capital elasticity, and thus are better off in the less talented country.

To further evaluate the role of asymmetric human capital distribution, elastic labor supply, non-homothetic preferences, and trade liberalization in determining outcomes at the aggregate, sectoral, and individual level, we run several counter factual exercises. The main findings are as follows: the more talented country outperforms the other one in terms of GDP and average living standards, due to larger market size. The less talented country in turn has a larger trade share, as more varieties are offered by the more talented country. Resource distribution becomes identical in both countries when either asymmetric distribution or non-homothetic preference structure is removed. When labor supply becomes exogenous, resource distribution tends to become more dispersed and inequality widens, because the elasticity of human capital against labor supply is less than 1 in the benchmark results. Trade balance is maintained within each sector both in the case of exogenous labor supply and homothetic preferences. Trade liberalization tends to reduce both within and between country inequality. When price levels become lower, individuals tend to increase their leisure time, and thus those with higher human capital will lose more income given the same magnitude of decline in labor supply, which explains the lower inequality within the country. The

shrinkage of between country inequality mainly occurs because when trade friction decreases, Country 1 can export to a larger market than Country 2. In addition, trade liberalization proportionally induces more firm entries into sectors in which Country 1 is a net exporter.

Our model is built upon the class of new trade models, such as Krugman [1980], and multiple variations and extensions of Melitz [2003]. However, these models typically assume that consumers have identical and homothetic preferences in all countries. As argued in Markusen [2013], such models imply that aggregate demand only depends on price indexes and aggregate income, and is independent of the distribution of income. We introduce non-homothetic preferences and heterogeneous individuals to an otherwise standard new trade model. The purpose is to evaluate the distributional impacts from changes in preference structure and trade liberalization. Our paper is most closely related to Matsuyama [2000], Matsuyama [2015] and Comin, Lashkari, and Mestieri [2015]. Matsuyama [2015] focuses on how non-homothetic preference structure may affect the trade pattern between two countries that differ only in population size. Individuals are homogeneous in his paper, and it is purely theoretical; simplification assumptions are required to obtain more analytical results. Comin, Lashkari, and Mestieri [2015] embed non-homothetic preferences into a multi-sector growth model to generate non-homothetic Engel curves at all levels of development.Duarte and Restuccia [2010] use a model with non-homothetic preferences to study structural transformation, and argue that non-homothetic preference is one reason for labor reallocation across sectors. Both households and firms are representative in their work, and the economy is closed and the role of trade liberalization is absent.

The major contribution of this paper is three fold. First, this theoretical framework is probably the first to embed non-homothetic CES preferences and heterogeneous individuals into a trade model with monopolistic competition. This framework allows us to explore the distributional impacts of non-homothetic preference structure and trade liberalization. Including leisure into the utility function also adds an extra term to human capital elasticity, which is thus lower than conventionally defined wage elasticity. Second, we introduce a two-tier utility structure with endogenous labor supply decision. The efficiency labor supply is then no longer equal to the population size. In addition, individual heterogeneity further introduces obstacles to analysis, because the market demand is now an integration of indi-

vidual demands of different types. These issues all greatly complicate the analysis compared with Matsuyama [2015], yet we are still able to obtain most analytical results even under a more general parameter space. Third, we go beyond Matsuyama [2015] by providing detailed quantitative analysis to quantify the role of each contributing factor in determining aggregate, sectoral, and individual-level outcomes. The findings shed light on the role of trade liberalization in within and between country inequality.

The rest of this paper is organized as follows. Section 2 describes the model setting. We perform quantitative analysis in Section 3. Our conclusion is offered in Section 4.

2 Model

The theoretical framework is an extension of Matsuyama [2015], with heterogeneous workers and endogenous labor supply decision. Moreover, we allow more flexible assumptions in the parameterization, but the analytical results still remain tractable. We start by characterizing the optimization problem for individuals and firms. We then define a closed-economy equilibrium and obtain some analytical properties. Finally, we open up the economy and examine how the patterns of trade are endogenously determined within the framework.

The economy is closed and populated with a mass N of workers. Workers are heterogeneous in term of their human capital endowment, which is assumed to follow a certain distribution function $G(\cdot)$. There are a finite number $S > 1$ of sectors in the economy.

2.1 Preferences and Individual Optimization Problem

Individual workers obtain utilities from final consumption as well as leisure. Specifically, the utility function for a worker with human capital h takes the following specific form:

$$U(h) = \frac{C(h)^{1-\sigma}}{1-\sigma} + \theta \frac{\ell(h)^{1-\sigma}}{1-\sigma},$$

where $C(h)$ denotes the final consumption and $\ell(h) \in (0, 1)$ is the leisure time, where we normalize the time endowment for each worker to be 1. $\theta > 0$ governs the preference of

leisure against final consumption.

We follow Matsuyama [2015] by assuming that final consumption is an implicit non-homothetic CES aggregator of sector-level consumption. Specifically, a worker with human capital h has final consumption $C(h)$ defined as follows:

$$\sum_{s=1}^{S} C\left(h\right)^{\frac{\epsilon(s)-\eta}{\eta}} C_s\left(h\right)^{\frac{\eta-1}{\eta}} = 1, \tag{1}$$

where $C_s(h)$ denotes sector-level consumption. When $\epsilon(s) = 1$, the functional form above can be reduced to standard homothetic CES preference. We formally characterize the optimization problem for a representative worker of human capital h as follows:

$$\begin{aligned} \max_{C_s(h),\ell(h)} \quad & \frac{C(h)^{1-\gamma}}{1-\gamma} + \theta \frac{\ell(h)^{1-\gamma}}{1-\gamma} \\ \text{s.t.} \quad & \sum_{s=1}^{S} C\left(h\right)^{\frac{\epsilon(s)-\eta}{\eta}} C_s\left(h\right)^{\frac{\eta-1}{\eta}} = 1, \\ & \sum_{s=1}^{S} C_s(h) P_s = E(h) \equiv wh(1-\ell(h)). \end{aligned}$$

The last line describes an individual's budget constraint, and P_s is the price level in sector s. w is the exogenous wage rate, and $h(1-\ell(h))$ is the efficiency labor supply. We solve the optimization problem above in the Appendix A.1. The sectoral consumption can be obtained as:

$$C_s\left(h\right) = \frac{E(h) P_s^{-\eta}}{\sum_t P_t^{1-\eta} C\left(h\right)^{\epsilon(t)-\epsilon(s)}}.$$

It is straightforward to show that consumption demand decreases with sectoral price and increases with expenditure level. By substituting $C_s(h)$ above into the expression of implicit consumption in equation 1, we have

$$E(h)^{1-\eta} = \sum_t P_t^{1-\eta} C(h)^{\epsilon(t)-\eta}. \tag{2}$$

Therefore, sectoral consumption can be further rewritten as

$$C_s(h) = \frac{E(h)^{\eta} C(h)^{\epsilon(s)-\eta}}{P_s^{\eta}}.$$

The elasticity of wage rate against the sectoral level consumption demand $C_s(h)$ can thus be expressed as:

$$\frac{\partial \ln C_s(h)}{\partial \ln w} = (\epsilon(s) - \eta)\frac{\partial \ln C(h)}{\partial \ln w} + \eta \frac{\partial \ln E(h)}{\partial \ln w}, \tag{3}$$

$$= (\varepsilon(s) - \eta)\frac{\sum_{t=1}^{S} P_t^{1-} C(h)^{\epsilon(t)-\eta}}{\sum_{t=1}^{S} \frac{\epsilon(t)-\eta}{1-\eta} P_t^{1-\eta} C(h)^{\epsilon(t)-\eta}} + \eta. \tag{4}$$

A number of features stand out from our setting. First, different from Matsuyama [2015] and others, income is no longer exogenous in our model, but instead depends on the labor supply decision made by the individual. Therefore, it is not suitable to define an income elasticity in this paper, and instead we define elasticity of human capital endowment against sectoral consumption demand in a later section. Second, if we rank sectors according to the value of $\epsilon(s)$ in ascending order, then sectors of a higher index have a higher value of ϵ. We restrict the analysis to the case where $0 < \epsilon(s) < \eta$ and $\eta > 1$. It is straightforward to show that the elasticity of the wage rate against sectoral-level consumption increases with the sector index. Moreover, the first part of equation 4 can be shown to be less than $1-\eta$ in the sector with the lowest index, and larger than $1-\eta$ in the sector with the highest index. Specifically, we have

$$\frac{\partial \ln C_1(h)}{\partial \ln w} < 1 \quad \text{and} \quad \frac{\partial \ln C_S(h)}{\partial \ln w} > 1.$$

The implication is that Sector 1 is the least wage-elastic sector, with elasticity less than one, and sector S is the most wage-elastic sector, with elasticity greater than one. Moreover, in any sector s, wage elasticity positively depends on the individual human capital level. That is, sectoral consumption is more elastic for individuals with higher human capital.

We can also show that sectoral price elasticity is independent of individual human capital

level and equal to η:

$$\frac{\partial \ln C_s(h)}{\partial \ln P_s} = \eta.$$

At a lower tier, sectoral consumption is a homothetic CES aggregator of all the varieties available in the sector. Specifically, the consumption level in sector s for an individual with human capital h, $C_s(h)$, can be expressed as:

$$C_s(h) = \Big[\int_{v \in \Omega_s} c_s(v,h)^{\frac{\sigma-1}{\sigma}} dv \Big]^{\frac{\sigma}{\sigma-1}},$$

where $c_s(v,h)$ denotes the consumption level for variety v within sector s for an individual with human capital h, and σ is the elasticity of substitution among all the varieties in sector s.

Conditional on the sectoral consumption demand $C_s(h)$ for an individual with human capital h, the demand for a single variety within sector s can thus be obtained by solving the following expenditure minimization problem:

$$\begin{aligned} &\min_{D_s(v,h)} \int_{v \in \Omega_s} p_s(v) c_s(v,h) dv \\ &\text{s.t.} \quad \Big[\int_{v \in \Omega_s} c_s(v,h)^{\frac{\sigma-1}{\sigma}} dv \Big]^{\frac{\sigma}{\sigma-1}} = C_s(h), \end{aligned}$$

where $p_s(v)$ is the price level for variety v in sector s. The solution yields:

$$c_s(v,h) = \frac{C_s(h)}{\int_{v \in \Omega_s} p_s(v)^{1-\sigma} dv} p_s(v)^{-\sigma}$$

We define the sectoral price index P_s as: [2]

$$P_s = \left[\int_{v \in \Omega_s} p_s(v)^{1-\sigma} dv \right]^{\frac{1}{1-\sigma}}.$$

2.2 Production

Production within each sector is characterized as monopolistic competition. Each firm is the single producer of a variety. Following Krugman [1980], firms are all identical within each sector. Labor is the only input of production. To produce one unit of variety in sector s requires ψ_s unit of labor. Free entry into each sector is assumed, and ϕ_s labor is required as the entry cost into sector s. Solving a monopoly firm's profit maximization problem by taking the individual worker's demand function as given yields:

$$p_s(v) = w\psi_s \frac{\sigma}{\sigma - 1}.$$

Free entry condition implies that each individual firm's sales revenue shall equal the sum of production and entry costs. That is:

$$p_s(v) D_s(v) - \psi_s D_s(v) w = w\phi_s.$$

$D_s(v)$ is the total demand for variety v, which is the sum of all individual workers' demands:

$$D_s(v) = N \int c_s(v,h) dG(h).$$

The quantity being produced for each variety in sector s can be solved from the free-entry

[2] The sector-level price level P_s is the cost to obtain one unit of sectoral output, which is essentially the solution to the following expenditure minimization problem:

$$\min \int_{v \in \Omega_s} p_s(v) c_s(v,h) dv$$
$$\text{s.t.} \quad \left[\int_{v \in \Omega_s} c_s(v,h)^{\frac{\sigma-1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}} = 1,$$

condition as:

$$D_s(v) = \frac{(\sigma - 1)\phi_s}{\psi_s}.$$

Total sectoral sales revenue thus equals $I_s w\sigma\phi_s$, where I_s denotes the number of firms in sector s. By equalizing above with the total expenditure on goods from sector s, we have:

$$I_s w\sigma\phi_s = N \int P_s C_s(h) dG(h).$$

The number of firms in sector s can thus be pinned down from the equation above.

2.3 Equilibrium in a Closed Economy

We restrict the analysis to a closed economy and summarize our equilibrium definition in this subsection.

Definition Given $\{\phi_s, \psi_s\}$ and wage rate w, the competitive equilibrium consists of a series of prices $\{p_s(v), P_s\}$, consumption quantities $\{C_s(h), c_s(v,h)\}$, the number of firms in each sector I_s, and the labor supply decision made by each individual $\ell(h)$, such that the following conditions hold:

1. Given $\{p_s(v), P_s\}$, each individual worker maximizes his utility by choosing $C_s(h), c_s(v,h)$ and $\ell(h)$.

2. Given the aggregate demand function from individual workers, each monopoly firm chooses the price of the variety to maximize its profit.

3. Free entry condition holds.

In Appendix A.1 solving the individual utility maximization problem can also yield the expression for leisure choice:

$$\ell(h) = C(h) \left[\frac{\sum_s (\epsilon(s) - \eta) C(h)^{\epsilon(s)-\eta-1} P_s^{1-\eta}}{wh(1-\eta)} \theta E(h)^{\eta} \right]^{\frac{1}{\gamma}} \tag{5}$$

We further explore the relation between human capital endowment and labor supply, in addition to total expenditure, in the following proposition.

Proposition 1 *In a closed economy, both leisure choice and expenditure level increase with human capital endowment. Moreover, the elasticity of human capital against labor supply or total expenditure is less than 1 when the following condition holds:*

$$\epsilon(S)^2 < \frac{\gamma - 1}{2}\eta$$

Proof *See the Appendix.*

If γ and η are high enough, which corresponds to a higher risk aversion coefficient or price elasticity, then the above inequality is more likely to hold. When γ and η are sufficiently large, it can be shown that both leisure and income($E(h)$) are increasing with the human capital level. As discussed before, because income is endogenous in our setting, we instead define an elasticity of human capital endowment against sectoral consumption as follows:

$$\frac{\partial \ln C_s(h)}{\partial \ln h} = \frac{\partial \ln C_s(h)}{\partial \ln E(h)} \frac{\partial \ln E(h)}{\partial \ln h} \tag{6}$$

$$= \underbrace{\left\{ (\varepsilon(s) - \eta) \frac{\sum_{s=1}^{S} P_s^{1-\eta} C(h)^{\epsilon(s)-\eta}}{\sum_{s=1}^{S} \frac{\epsilon(s)-\eta}{1-\eta} P_s^{1-\eta} C(h)^{\epsilon(s)-\eta}} + \eta \right\}}_{\text{wage elasticity}} * \underbrace{\left\{ 1 - \frac{\ell(h)}{1-\ell(h)} \frac{\partial \ln \ell(h)}{\partial \ln h} \right\}}_{\text{elasticity of human capital against labor supply}} . \tag{7}$$

We can decompose human capital elasticity as defined above into two parts: wage elasticity and the elasticity of human capital against labor supply. Note that when leisure is not included in the utility function ($\theta = 0$), our expression can be reduced to wage elasticity. In addition, when $\epsilon(s)$ is no longer sector-specific, the wage elasticity term reduces to one and thus the elasticity of human capital against labor supply dominates. From Proposition 1, we have already proved that when η and γ are sufficiently large, labor elasticity is within the range 0 to 1. Therefore, human capital elasticity is smaller than wage elasticity. In addition, we have shown earlier that wage elasticity against consumption demand for sector S is larger than 1, which may no longer be true for human capital elasticity after taking into account

the effects of endogenous labor supply. This result implies that consumption demand is less responsive to changes in human capital than in wage levels.

2.4 Trade Equilibrium and Patterns of Trade

We apply the setting to an open economy with two countries, Country 1 and 2. The two countries are identical in every aspect except for the initial human capital distribution. We let the distribution in Country 2 first-order stochastically dominate that of Country 1. Therefore, Country 2 is denoted as a more talented economy, as it comprises more individuals with higher human capital.

Standard iceberg trade cost assumption applies. To deliver 1 unit of goods from country j to i, $\tau_{ij} > 1$ units of goods need to be shipped. Country $j's$ demand for a variety produced in sector s of country i can thus be solved as:

$$D_s^{ij}(v) = N^j \frac{\int_{\underline{h}}^{\overline{h}} E^j(h)^{\eta} C^j(h)^{\varepsilon^j(s)-\eta} dG(h)}{P_s^{j\eta-\sigma}} p_s^{ij}(v)^{-\sigma}.$$

The price for a variety sold to country j from sector s of country i is thus:

$$p_s^{ij}(v) = \tau_{ij} w^i \psi_s^i \frac{\sigma}{\sigma - 1}.$$

Moreover, the free-entry condition in country j implies:

$$\sum_i \left[p_s^{ji}(v) q_s^{ji}(v) - \tau_{ji} \psi_s^j q_s^j(v) w^j \right] = w^j \phi_s^j.$$

In a closed economy, the wage rate is assumed to be exogenous. In the case of an open economy, the relative wage between the two countries will be pinned down by the trade balance in each country:

$$w^j L^j = \sum_{i=1}^{J} \frac{\sum_s V_s^j D_s^{ji}(v) p_s^{ji}(v)}{\sum_{j=1}^{J} \sum_s V_s^j D_s^{ji}(v) p_s^{ji}(v)} w^i L^i, \quad j \in \{1, 2\}$$

The net export value in sector s of country i is the value of goods sold to country j minus

the value of goods purchased from country j, which can be expressed as

$$NX_s^i = V_s^i D_s^{ij}(v) p_s^{ij}(v) - V_s^j D_s^{ji}(v) p_s^{ji}(v). \tag{8}$$

We denote m_s^i to be the expenditure share on sector s in country i, and L_i^e to be the total efficiency labor supply in country i. These can be expressed as:

$$m_s^i = \frac{\int E_s^i(h)\, dG^i(h)}{\int E^i(h)\, dG^i(h)} = \frac{\int E^i(h)^{\eta} C^i(h)^{\varepsilon(s)-\eta} (P_s^i)^{1-\eta} dG^i(h)}{\int E^i(h)\, dG^i(h)}, \tag{9}$$

$$L_i^e = N_i \int h(1-\ell(h)) dG^i(h). \tag{10}$$

In the Appendix A.3, we show in detail that the general formula for the net export value in country i can be expressed as:

$$NX_s^i = \rho L_j^e w^j \left\{ \frac{m_s^i \omega}{\omega^\sigma - \rho} \frac{L_i^e}{L_j^e} - \frac{\omega^\sigma m_s^j}{1 - \rho\omega^\sigma} \right\}, \tag{11}$$

where $\omega = \frac{w^i}{w^j}$ and $\rho = \tau_{ij}^{1-\sigma}$.

When there are no sectoral differences in the labor requirement and entry cost, the expression above can be further reduced to the one in Matsuyama [2015]:

$$NX_s^i = \frac{\rho L_i^e w^i}{\omega^\sigma - \rho} \left(m_s^i - m_s^j \right)$$

By evaluating equation 11, it is straightforward to demonstrate that changes in the following variables can contribute to a higher net export value:

- higher m_s^i or lower m_s^j;
- lower trade cost τ;
- higher ratio in the efficiency labor supply L_i^e / L_j^e.

Due to the non-homothetic preference structure, individuals with higher human capital tend to proportionally spend more on sectors with a higher index. Moreover, together with

the home market effects proposed in Krugman [1980], these results imply that the more talented country tends to be a net exporter of goods from the sector with higher index.

3 Quantitative Analysis

We quantify the model in this section. We restrict the analysis to a two-country setting, where countries are identical in every aspect except for the distribution of the initial human capital endowment. The purpose is to examine how non-homothetic preference structure, the asymmetry of human capital distribution, endogenous labor supply, and trade liberalization affect the economy from different perspectives. At the aggregate level, we focus on the impacts on GDP, income inequality as well as terms of trade for both countries. At the sectoral level, we focus on the distribution of employment, number of firms, and sales revenue across sectors. Finally, at the individual level, we explore how individuals at different percentiles of human capital distribution are affected in term of their consumption, income, utilities, etc.

3.1 Parameterization

We calibrate the model into a closed economy. We let human capital distribution follow a Pareto distribution defined over $[\underline{h}, \infty)$:

$$G(h) = 1 - (\frac{h}{\underline{h}})^{\alpha},$$

where $\alpha > 0$ governs the skewness of the distribution. The parameter setting in the case of two countries only differs from the closed economy in the distribution of initial human capital and iceberg trade cost, which will be specified later in this section. We let the number of sectors be three. The data counterpart refers to the agriculture, manufacturing and service sector, respectively. We normalize the population size to be 1. γ governs the risk-aversion coefficient, and we follow the literature in setting it at 2. σ captures sectoral-level price elasticity. We assume σ is larger than η to mimic the fact that differentiated goods are closer substitutes within each sector than across sectors. We let σ equal 6 and η equal 4,

so they are consistent with the range commonly used in the literature. We normalize the labor requirement in the manufacturing sector ψ_2 to be 1. Similarly, we also normalize the entry cost in the manufacturing sector ϕ_2 to be 1. The minimum human capital level ($\underline{h}$) is set at 1. The remaining parameters include $\{\psi_1, \psi_3, \phi_1, \phi_3, \epsilon_1, \epsilon_2, \epsilon_3, \alpha, \theta\}$. We calibrate them to simulate a hypothetic economy with the following distinctive features. ψ_1 and ψ_3 are the unit labor requirement for the agricultural and service sectors, respectively. We calibrate them to match an equal employment share (1/3) in each sector. ϕ_1 and ϕ_3 are the entry costs in term of labor requirement in the agricultural and service sectors, respectively. Similarly, we calibrate them to match an equal fraction (1/3) of firms in each sector. As shown before, $\epsilon_i (i = 1, 2, 3)$ reflects the wage elasticity of sector i. We follow Markusen [2013] to target a wage elasticity of 0.65 in the agriculture sector, 1.0 in manufacturing sector, and 1.7 in service sector. Finally, θ controls the preference for consumption against leisure, and is chosen to match a 1/3 fraction of leisure time on average among all agents. α is the shape parameter of the Pareto distribution. We have it match a ratio of 3.2 between the 90th and 10th percentile of the income distribution.

In the two-country case, we first let the human capital distribution of Country 2 first-order stochastically dominate that of Country 1, which implies Country 2 is a more talented country. We set the shape parameter (α) in Country 2 to be 20 percent lower than that in Country 1, while keeping the scale parameter ($\underline{h}$) the same between the two countries. Secondly, we calibrate the iceberg trade cost parameter τ to match an average 30 percent of trade share. We summarize the calibrated parameter values in Table 1.

Given the benchmark parameterization, we numerically solve the model. The algorithm involves first obtaining demand at the upper tier: given the wage rate and sectoral prices, we solve for sectoral consumption and leisure choice. At the lower tier, given the sectoral consumption, we solve for demand against each variety. On the production side, we solve for firm distribution across sectors using free-entry conditions. Finally, market clearing conditions pin down wages and sectoral prices. The benchmark results are reported in Table 2.[3]

[3]Note that the benchmark results differ from the calibration targets in the distribution of employment and firm numbers because we now open up the economy.

Para.	Sources/Targets	Para. Value
ψ_1	employment share in agriculture sector	0.78
ψ_2	normalization	1
ψ_3	employment share in service sector	1.04
ϕ_1	number of firm share in agriculture sector	1.45
ϕ_2	normalization	1
ϕ_3	number of firm share in service sector	0.46
ϵ_1	wage elasticity in agriculture sector	0.5
ϵ_2	wage elasticity in manufacture sector	1.25
ϵ_3	income elasticity in service sector	1.90
α	income 90/10 ratio	2.02
θ	share of leisure time	0.16
σ	elasticity of substitution within sectors	6
η	price elasticity	4
γ	risk aversion coefficient	2

Table 1: Benchmark Parameterizations

Note: We calibrate the sectoral labor requirement and entry fee to mimic an even distribution of employment and the number of firms across sectors in a closed economy. The elasticity of substitution is chosen to match the income elasticity of agricultural, manufacturing, and service goods estimated in the literature. The shape parameter in the less talented country is calibrated to mimic a 90-10 income ratio at 3.2. The iceberg trade is chosen to match an average 30 percent of trade share in both countries. Other parameters we choose either by normalization or by following the literature. For more details, see the main text.

In the top panel, we present the aggregate results. Both GDP and per-capita GDP in Country 2 are 1.17 times of those in Country 1. The main reason is that Country 2 is a more talented country with higher human capital stock, and thus the efficiency labor supply is more ample. These features push up the total labor income, which equals to GDP in this setting due to the free entry condition. As the population size is equalized in both countries, higher GDP necessarily results in higher per-capita GDP. If we define the trade share as the fraction of expenditure on imported goods, our results suggest that trade share in Country 1 is larger than in Country 2. This difference is mainly driven by the larger market size of Country 2. As a result of the home market effect, more varieties are produced in Country 2 than in Country 1. Individuals in Country 1 thus spend proportionally more on goods produced in Country 2. We measure inequality using the coefficient of variation in individual income and consumption, as well as the 90-10 income ratio. Because human capital distribution is more dispersed in Country 2 than Country 1, it is not surprising to

	GDP	Trade Share	CV_Inc.	CV_Cons.	9010 Inc.R.
Country 1	0.995	0.322	0.352	0.438	2.268
Country 2	1.000	0.275	0.376	0.466	3.209

	Country 1			Country 2		
	Sec 1	Sec 2	Sec 3	Sec 1	Sec 2	Sec 3
Employment	0.245	0.316	0.439	0.157	0.284	0.559
Expenditure	0.217	0.310	0.473	0.182	0.297	0.521
Firm Dist	0.248	0.308	0.445	0.159	0.276	0.566
Trade Share	0.322	0.322	0.322	0.275	0.275	0.275
	Country 1			Country 2		
	Bottom	Median	Top	Bottom	Median	Top
Utility	-1.336	-1.073	-0.636	-1.423	-0.985	-0.506
Consumption	1.200	1.603	3.413	1.107	1.804	4.964
Leisure	0.324	0.362	0.475	0.314	0.378	0.535
Income	0.734	0.923	1.664	0.686	1.009	2.203

Table 2: Benchmark Results

Note: In the top panel, we present benchmark results for the aggregate variables including GDP and trade share, as well as inequality measures in income and consumption. In the middle panel, we present benchmark results on sectoral distribution in terms of employment, expenditure, and firm numbers. We also report sectoral prices and net exports. In the bottom panel, we select individuals with human capital in the bottom 10, median and top 10 percentile of the distribution in each country, and compute their utilities, consumption, leisure, and income levels. For more detail, see the main text.

find that Country 2 exhibits larger inequality in both income and consumption.

In the middle panel, we report the sectoral results. Employment share and the fraction of firms in Sector 3 dominate in both countries. Country 1 has a higher (lower) employment share and fraction of firms in Sectors 1 and 2 (3), while expenditure share for Sector 3 appears to be the highest in Country 2. The simulation results are consistent with our theoretical predictions. Because Sector 3 is the one with the highest human capital elasticity, and human capital distribution in Country 2 has a thicker right tail than that of Country 1, it would seem that the demand for varieties produced in Sector 3 is higher in Country 2 than Country 1. Due to the home market effect, Country 2 should have a larger Sector 3. The second to last row also reports the net export value in each sector. Trade balance is maintained within each country but not within each sector. Our results suggest that Country 1 is a net exporter of goods produced in Sectors 1 and 2, while Country 2 is a net exporter of goods

produced in Sector 3.

In the bottom panel, we select individuals with human capital endowment in the bottom 10 percentile, median and top 10 percentile of the distribution in each country. It is interesting that individuals at the bottom of the distribution enjoy higher utilities and consumption in Country 1, whereas the pattern is reversed at the median and top percentile of the distribution. [4] These results are largely due to the fact that the price index in Sector 1 is lower in Country 1 than in Country 2. Individuals in the bottom percentile of the human capital distribution proportionally spend more on Sector 1 than on other sectors and thus constitute the major demand for goods produced in Sector 1. Country 1 has more individuals at the lower end of the distribution, which also explains why Country 1 has more labor and firms in Sector 1 than Country 2. Therefore, the sectoral price in Sector 1 should be lower in Country 1. In addition, because Country 1 is a major exporter of goods produced in Sector 1, individuals from Country 2 need to pay the extra iceberg trade cost to import from Country 1, which further lowers their consumption level and utilities.

3.2 Counterfactual Exercises

To further evaluate the role of asymmetric human capital distribution, endogenous labor supply decision, non-homothetic preferences, and trade in determining outcomes at aggregate, sectoral and individual levels, we perform several counter factual exercises in this subsection by either setting the labor supply to be exogenous, setting preferences to be homothetic, or letting the human capital distribution in the two countries be identical.

Symmetric Distribution If we let human capital distribution in Country 2 be identical to that of Country 1, Country 2 then becomes less talented than its benchmark counterpart. It is thus not surprising to observe that GDP in Country 1 slightly increases, while GDP in Country 2 declines by 14.3 percent relative to its benchmark level as shown in Table 3.

[4] Since the human capital distribution are different in the two countries, and thus the corresponding human capital levels at each percentile are different in the two countries. The human capital level for individual at bottom 10 percentile in country 1 is higher than that in country 2. However, the results that individuals at the bottom percentile obtain higher utilities than those in country 2 still remain even though we focus on the individuals with the lowest human capital, which are the same in the two countries.

	GDP	Trade Share	CV_Inc.	CV_Cons.	9010 Inc.R.
			Bechmark		
Country 1	106.073	0.322	0.352	0.438	2.268
Country 2	124.477	0.275	0.376	0.466	3.209
			Symmetric Distribution		
Country 1	106.660	0.298	0.353	0.438	2.269
Country 2	106.660	0.298	0.353	0.438	2.269
			Homothetic Preference		
Country 1	105.763	0.322	0.355	0.421	2.267
Country 2	124.329	0.274	0.382	0.448	3.217
			Exogenous Labor Supply		
Country 1	184.072	0.344	0.480	0.606	2.917
Country 2	247.174	0.256	0.534	0.671	4.735
			Autarky		
Country 1	107.634	0.000	0.354	0.439	2.277
Country 2	125.638	0.000	0.377	0.467	3.225

Table 3: Counter-factual Results: aggregate

Note: We compare aggregate results from the benchmark economy with several counter factual economies, including symmetric human capital distribution, exogenous labor supply, homothetic preferences and an autarky case. For more details, see the main text.

The trade share in both countries has declined. When human capital distribution becomes symmetric, the two countries are essentially identical in every aspect. Hence, trade balance is expected within each sector, as is shown in Table 4. Such a trade balance may in turn lower the aggregate trade share. In term of inequality measures, Country 2 has seen a decline in both income and consumption inequality as a result of less dispersed human capital distribution, while inequality measures in Country 1 remain stable relative to benchmark levels. In addition, when there are fewer talented individuals in Country 2, the non-homothetic preference structure ensures that the demand for varieties in Sector 3 declines the most among the sectors in Country 2. Therefore, Sector 3 in Country 2 has greatly shrunk as shown in Table 4. Moreover, Table 4 suggests that the employment share and the fraction of firms in Sectors 1 and 2 have shrunk in Country 1 and expanded in Country 2. The expenditure share does not change much in Country 1, while less is spent in Country 2 on Sector 3 to

	Country 1			Country 2		
	Sector 1	Sector 2	Sector 3	Sector 1	Sector 2	Sector 3
	Bechmark					
Employment	0.245	0.316	0.439	0.157	0.284	0.559
Expenditure	0.217	0.310	0.473	0.182	0.297	0.521
Firm Dist	0.248	0.308	0.445	0.159	0.276	0.566
Net Export	2.998	1.117	-4.114	-2.998	-1.117	4.114
Prices	0.954	0.927	0.872	0.968	0.927	0.858
	Symmetric Distribution					
Employment	0.217	0.306	0.477	0.217	0.306	0.477
Expenditure	0.218	0.310	0.472	0.218	0.310	0.472
Firm Dist	0.219	0.297	0.483	0.219	0.297	0.483
Net Export	0.000	0.000	0.000	0.000	0.000	0.000
Prices	0.961	0.935	0.879	0.961	0.935	0.879
	Homothetic Preference					
Employment	0.337	0.316	0.347	0.337	0.316	0.347
Expenditure	0.338	0.320	0.343	0.338	0.320	0.343
Firm Dist	0.341	0.307	0.351	0.341	0.307	0.351
Net Export	0.000	0.000	0.000	-0.000	-0.000	-0.000
Prices	0.908	0.925	0.904	0.903	0.920	0.899
	Exogenous Labor Supply					
Employment	0.149	0.298	0.553	0.056	0.199	0.745
Expenditure	0.118	0.269	0.614	0.080	0.229	0.691
Firm Dist	0.151	0.290	0.560	0.056	0.193	0.751
Net Export	5.887	6.300	-12.186	-5.887	-6.300	12.186
Prices	0.953	0.880	0.790	0.985	0.887	0.773
	Autarky					
Employment	0.222	0.307	0.471	0.185	0.294	0.521
Expenditure	0.223	0.311	0.466	0.186	0.298	0.516
Firm Dist	0.225	0.298	0.477	0.187	0.285	0.528
Net Export	0.000	-0.000	-0.000	-0.000	0.000	0.000
Prices	0.996	0.972	0.915	0.999	0.960	0.889

Table 4: Counter-factual Results: sector

Note: We compare sectoral-level results from the benchmark economy with several counter factual economies, including symmetric human capital distribution, exogenous labor supply, homothetic preferences and an autarky case. For more details, see the main text.

compensate for higher expenditure on other sectors.

At the individual level, because the two countries become symmetric, the selected indi-

	Country 1			Country 2		
	Bottom	Median	Top	Bottom	Median	Top
	Bechmark					
Utility	-1.431	-1.073	-0.636	-1.423	-0.985	-0.506
Consumption	1.098	1.603	3.413	1.107	1.804	4.964
Leisure	0.313	0.362	0.475	0.314	0.378	0.535
Income	0.684	0.923	1.664	0.686	1.009	2.203
	Symmetric Distribution					
Utility	-1.435	-1.076	-0.638	-1.435	-1.076	-0.638
Consumption	1.094	1.597	3.400	1.094	1.597	3.400
Leisure	0.313	0.362	0.474	0.313	0.362	0.474
Income	0.687	0.928	1.674	0.687	0.928	1.674
	Homothetic Preference					
Utility	-1.428	-1.077	-0.647	-1.417	-0.989	-0.518
Consumption	1.097	1.586	3.280	1.108	1.779	4.674
Leisure	0.315	0.365	0.476	0.316	0.381	0.536
Income	0.682	0.919	1.658	0.684	1.004	2.199
	Exogenous Labor Supply					
Utility	-0.525	-0.323	-0.114	-0.517	-0.274	-0.065
Consumption	1.905	3.100	8.746	1.934	3.650	15.375
Leisure	0.000	0.000	0.000	0.000	0.000	0.000
Income	0.992	1.442	3.155	1.000	1.623	4.735
	Autarky					
Utility	-1.479	-1.107	-0.653	-1.462	-1.010	-0.515
Consumption	1.052	1.537	3.280	1.068	1.743	4.811
Leisure	0.308	0.357	0.468	0.309	0.373	0.530
Income	0.692	0.936	1.692	0.691	1.017	2.227

Table 5: Counter-factual Results: individual

Note: We compare individual-level results from the benchmark economy with several counter factual economies, including symmetric human capital distribution, exogenous labor supply, homothetic preferences and an autarky case. For more details, see the main text.

viduals thus obtain the same levels of consumption, income, and utilities in each country. Note that it is not a fair comparison between the benchmark and the counter-economy for selected individuals in Country 2; the cutoff human capital levels at each percentile vary when the human capital distribution changes, so we are not comparing the same individual in both economies. Therefore, we only focus on results for individuals in Country 1. In

comparison with the benchmark economy, utilities and consumption levels become lower for all individuals. This result is due to higher price levels, as shown in Table 4, because of less efficiency labor supply and smaller market demand. The only exception stems from the higher income levels for individuals at the median and top-10 percentile of the distribution compared to the benchmark counterpart, likely because higher price levels induce less leisure and higher nominal wage in the country. However, despite rising incomes, living standards still worsen in the counterfactual economy.

Non-homothetic Preferences We examine the role of non-homothetic preferences by looking into an alternative economy, where $\epsilon(s)$ is identical across all sectors and takes the average value among their benchmark counterparts. Table 3 suggests that aggregate statistics such as GDP and trade share do not differ much from the benchmark levels. Under homothetic preferences, human capital elasticity no longer increases with the sector index. Therefore, individuals with higher human capital do not proportionally spend more on goods from sectors with a higher index. As a result, the distribution of consumption become more even and less dependent on the human capital distribution. At the sectoral level, when human capital elasticity becomes independent from the sector index, the distributional impacts on sectoral consumption demand are completely eliminated. Similar to those trade models with homothetic CES preferences and monopolistic competition, only aggregate expenditure matters in determining sectoral demand. Because sectors are only different in their unit labor requirement and entry cost, and are identical in both countries, as a result, homothetic preference structure entails an identical resource distribution between the two countries, as shown in Table 4. In addition, employment, number of firms and the expenditure become almost evenly distributed across sectors in the counter factual economy. This result implies that non-homothetic preference structure greatly contributes to the dispersion of resource distribution across sectors. Note that the ranking in the size of each sector also changes, with Sector 3 still being the largest, but followed by Sector 1 and 2. In all other counter factual exercises, the ranking remains the same as in the benchmark economy. Trade balance is also maintained within each sector.

When human capital elasticity does not depend on the sector index, Country 2 has larger

market demand for each sector, and thus sectoral-level prices are all cheaper in Country 2 than in Country 1, as shown in Table 4. This result explains why individuals at the bottom percentile of the distribution from Country 1 are no longer better off than those from Country 2, as shown in Table 5. Comparing price levels with their benchmark counterparts, it can be seen that price levels become higher in Sector 3 and lower in the remaining sectors. This result again is due to the reduction in the overall demand for Sector 3, because of the shift in preference structure. As expenditure share does not vary over individual's human capital level[5], the uneven changes in sectoral price levels may exert an uneven impact on each individual's leisure choice: individuals at the bottom may proportionally spend more on Sector 3 than in the benchmark economy, and so they need to reduce their leisure time to overcome the higher price level in Sector 3. In contrast, individuals at the median and top part of the distribution may manage to increase their leisure time and thus may not need to spend as much as in the benchmark case on Sector 3, plus the price levels in Sector 1 and Sector 2 also become cheaper. This argument is supported by Table 4.

Endogenous Labor Supply We shut down the endogenous labor supply decision by setting θ equal to zero, and thus each individual inelastically supplies one unit of labor. Table 3 suggests that all aggregate statistics become significantly higher than in the benchmark counterpart. When leisure is not considered, the efficiency labor supply in each country greatly increases, which results in higher GDP. Our theoretical results predict that labor supply decreases with the human capital level. That is, in the benchmark economy, more talented individuals enjoy more leisure time. In contrast, when each individual instead inelastically supplies one unit of labor in the counter-economy, inequality in both income and consumption will greatly increase. At the sectoral level, Sector 3 becomes much larger in terms of employment, expenditure, and the number of firms in the counter-economy. The results are supported by our expression of human capital elasticity derived in Equation (7). In the benchmark economy, the elasticity of human capital against leisure choice is between

[5]In the case of CES preferences, the sectoral expenditure for any individual becomes:

$$m_s = \frac{P_s^{1-\eta}}{\sum_t p_t^{1-\eta}}$$

0 and 1, which causes the human capital elasticity for each sector to be lower than in the case of exogenous labor supply. Moreover, when labor supply is exogenous, the elasticity of human capital against consumption demand equals wage elasticity, and is proved to be larger than 1 in Sector 3. Therefore, Sector 3 becomes more dominant in resource distribution. In terms of trade pattern, Country 1 is still the net exporter of goods produced from Sectors 1 and 2. The trade volume becomes significantly higher than in the benchmark economy due to more efficiency labor supply and thus larger market sizes.

Despite lower price levels in Sectors 1 and 2 of Country 1, individuals at the bottom percentile of the distribution from Country 1 become worse off than those in Country 2, which is different from the findings in the benchmark economy. This result occurs because a majority of the expenditure share is devoted to Sector 3, whose price level is lower in Country 2. It may also be partially due to the lack of a trade-off between leisure and consumption. In the benchmark results, individuals at the bottom of the distribution in Country 1 are found to enjoy more leisure time than those in Country 2.

Autarky In this exercise, we evaluate the role of trade liberalization. We let the parameter of iceberg trade cost increase to a sufficiently high value, so that the trade share in both countries is close to zero. When trade is not allowed, the number of varieties available in both countries will significantly drop, and thus the price index increases. Our results show that nominal GDP appears to be higher than in the benchmark case, likely because individuals tend to provide more labor in the event of a higher price index. However, this result does not necessarily imply a higher living standard, as shown in Table 5. Table 4 suggests more labor is reallocated from Sector 1 to Sector 3 of Country 2 when trade friction decreases. The exactly opposite pattern is observed in Country 1, which suggests Country 2 becomes more specialized in Sector 3, and Country 1 more specialized in Sector 1, in the process of trade liberalization.

To examine the role of trade liberalization in within and between country inequality, our results in Table 3 suggest that trade liberalization tends to reduce consumption and income inequality within each country. When price levels become lower, individuals wish to increase their leisure time, and thus those with higher human capital will lose more income given the

same magnitude of decline in labor supply. This result explains the lower inequality within the country.

Our results in Table 5 also indicate that trade liberalization tends to shrink the gap in living standards between the countries. In the autarky case, the relative utilities between the two countries for individuals at the top 10 percentile of the distribution is $-0.515/(-0.653) = 0.789$, and the number becomes 0.912 and 0.989 for individuals at the median and bottom of the distribution. In the benchmark economy, those ratios all become closer to 1 at 0.796, 0.918 and 0.992, respectively. Moreover, a further look at the utility gain for different individuals suggests the following: from autarky to the benchmark economy, utilities for individuals at the top 10 percentile of the distribution increase by 2.6 percent in Country 2, compared with 1.75 percent in Country 1. In contrast, utilities for those at the bottom of the distribution increase by 2.67 percent in Country 1, compared with 2.16 percent in Country 2. That is, the more talented individuals benefit more from staying in Country 2, and less talented ones benefit more from staying in Country 1. Moreover, the bottom ones benefit proportionally more than talented individuals in both countries, which may provide further evidence for the decline of the within-country inequality.

Two major sources contribute to the shrinking gap between the countries: 1) Country 1 gains access to a larger market in the event of trade liberalization than Country 2;. 2) Country 1 is a net exporter of Sectors 1 and 2, while Country 2 is a net exporter of Sector 3. More than half of the total expenditure in the economy is still devoted to Sectors 1 and 2. Therefore, trade liberalization proportionally induces more firm entries into Sector 1 of Country 1 than into Sector 3 of Country 2. As a result, the price index in Sector 1 in Country 1 should proportionally decrease more. As shown in Table 4 in the benchmark economy, the price level of Sector 3 in Country 2 is 96.51 percent of that in autarky, while the number becomes 95.78 percent for Sector 1 in Country 1. Therefore, as the major exporter of Sector 1 and 2, Country 1 benefits more from trade liberalization than Country 2.

4 Conclusion

In this paper, we have developed a trade model with monopolistic competition, non-homothetic preference structure, heterogeneous agents, and endogenous labor supply. Individuals differ in their human capital endowment, and the two countries are identical except for the distribution of human capital. We apply the theoretical framework to quantitatively examine how preference structure, elastic labor supply, asymmetry in distribution and trade liberalization affect economic performance at the aggregate, sectoral, and individual level. The major findings are as follows: the more talented country outperforms the other in terms of GDP and average living standards, due to a larger market size. However, individuals from the bottom percentile of the distribution may be better off than those from the more talented country. The resource distribution becomes identical in both countries when either asymmetric distribution or non-homothetic preferences are removed. When labor supply becomes exogenous, resource distribution tends to become more dispersed and inequality widens. Trade liberalization tends to reduce inequality within and between countries.

References

Blackorby, C., and R. R. Russell (1981): "The Morishima elasticity of substitution; symmetry, constancy, separability, and its relationship to the Hicks and Allen elasticities," *The Review of Economic Studies*, 48(1), 147–158.

Buera, F. J., and J. P. Kaboski (2009): "Can traditional theories of structural change fit the data?," *Journal of the European Economic Association*, 7(2-3), 469–477.

Comin, D., D. Lashkari, and M. Mestieri (2015): "Structural Change with Long-run Income and Price Effects," Discussion paper.

Duarte, M., and D. Restuccia (2010): "The role of the structural transformation in aggregate productivity," *The Quarterly Journal of Economics*, 125(1), 129–173.

Eaton, J., and S. Kortum (2002): "Technology, geography, and trade," *Econometrica*, 70(5), 1741–1779.

Krugman, P. (1980): "Scale economies, product differentiation, and the pattern of trade," *The American Economic Review*, 70(5), 950–959.

Markusen, J. R. (2013): "Putting per-capita income back into trade theory," *Journal of International Economics*, 90(2), 255–265.

Matsuyama, K. (2000): "A ricardian model with a continuum of goods under nonhomothetic preferences: Demand complementarities, income distribution, and north-south trade," *Journal of Political Economy*, 108(6), 1093–1120.

——— (2015): "The Home Market Effect and Patterns of Trade Between Rich and Poor Countries," Discussion paper.

Melitz, M. J. (2003): "The impact of trade on intra-industry reallocations and aggregate industry productivity," *Econometrica*, 71(6), 1695–1725.

A Appendix

A.1 Solving the Individual Optimization Problem

An individual worker of human capital h solves the following optimization problem:

$$\max \frac{C(h)^{1-\gamma}}{1-\gamma} + \theta \frac{\ell(h)^{1-\gamma}}{1-\gamma}$$
$$s.t \quad \sum p_s C_s(h) = wh(1-\ell(h))$$

We set up the following Langrange:

$$L = \frac{C(h)^{1-\gamma}}{1-\gamma} + \theta \frac{\ell(h)^{1-\gamma}}{1-\gamma} + \lambda \left[wh(1-\ell(h)) - \sum p_s C_s(h) \right]$$

Taking first order condition with respect to C_s and ℓ gives:

$$\begin{aligned} C(h)^{-\gamma} \frac{\partial C(h)}{\partial C_s(h)} &= \lambda p_s, \forall s \\ \theta \ell(h)^{-\gamma} &= \lambda. \end{aligned}$$

We differentiate $C_s(h)$ from the recursive utility specification,

$$\sum_t C(h)^{\frac{\epsilon(t)-\eta}{\eta}} C_t(h)^{\frac{\eta-1}{\eta}} = 1,$$

and thus

$$\begin{aligned}\frac{\partial C(h)}{\partial C_s(h)} &= \frac{C(h)^{\frac{\epsilon(s)-\eta}{\eta}} \frac{1-\eta}{\eta} C_s(h)^{\frac{-1}{\eta}}}{\sum_t \frac{\epsilon(t)-\eta}{\eta} C(h)^{\frac{\epsilon(t)-2\eta}{\eta}} C_t(h)^{\frac{\eta-1}{\eta}}} \\ &= \frac{(1-\eta) C(h)^{\frac{\epsilon(s)-\eta}{\eta}} C_s^j(h)^{\frac{-1}{\eta}}}{\sum_t (\epsilon(t)-\eta) C(h)^{\frac{\epsilon(t)-2\eta}{\eta}} C_t^j(h)^{\frac{\eta-1}{\eta}}}\end{aligned}$$

The first order conditions thus become:

$$C(h)^{-\gamma} \frac{(1-\eta) C(h)^{\frac{\epsilon(s)-\eta}{\eta}} C_s(h)^{\frac{-1}{\eta}}}{\sum_t (\epsilon(t)-\eta) C(h)^{\frac{\epsilon(t)-2\eta}{\eta}} C_t(h)^{\frac{\eta-1}{\eta}}} = \frac{\theta \ell(h)^{-\gamma}}{wh} p_s, \forall s \tag{12}$$

Taking the ratio of two arbitrary first order conditions with respect to C_s and C_t gives:

$$C(h)^{\frac{\epsilon(s)-\epsilon(t)}{\eta}} \left(\frac{C_s(h)}{C_t(h)}\right)^{\frac{-1}{\eta}} = \frac{p_s}{p_t}$$

that is,

$$\frac{C_t(h)}{C_s(h)} = \left[\frac{p_s}{p_t} C(h)^{\frac{\epsilon(t)-\epsilon(s)}{\eta}}\right]^{\eta}$$

.

Substituting above into the budget constraint gives:

$$\sum_t p_t \left[\frac{p_s}{p_t} C(h)^{\frac{\epsilon(t)-\epsilon(s)}{\eta}}\right]^{\eta} C_s(h) = wh(1-\ell(h))$$

Therefore, the sectoral-level consumption can be obtained as:

$$C_s(h) = \frac{wh(1-\ell(h))}{\sum_t p_t^{1-\eta} p_s^{\eta} C(h)^{\epsilon(t)-\epsilon(s)}} = \frac{E(h) P_s^{-\eta} C_s(h)^{\epsilon(s)-\eta}}{\sum_t P_t^{1-\eta} C(h)^{\epsilon(t)-\eta}}.$$

The expression for leisure choice can be obtained as:

$$\frac{C(h)}{\ell(h)} = \left[\frac{\sum_s [\varepsilon(s)-\eta] C(h)^{\varepsilon(s)-\eta} (P_s)^{1-\eta}}{wh(1-\eta)} \theta C(h)^{-1} E(h)^{\eta}\right]^{-\frac{1}{\gamma}}.$$

A.2 The elasticity of Human capital against Sectoral-level Consumption

Substituting the expression for $C_s(h)$ into the recursive utility specification leads to:

$$\sum_s C(h)^{\frac{\varepsilon(s)-\eta}{\eta}} \frac{E^{\frac{\eta-1}{\eta}} \left(C(h)^{\varepsilon(s)-\eta}\right)^{\frac{\eta-1}{\eta}}}{P_s^{\eta-1} \left(\sum_t P_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}\right)^{\frac{\eta-1}{\eta}}} = 1$$

As a result,

$$E(h)^{1-\eta} \equiv \sum_t P_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}. \tag{13}$$

Substituting the expression of $E(h)$ into the expression of $C_s(h)$ yields:

$$C_s(h) = \frac{E(h) C(h)^{\varepsilon(s)-\eta}}{P_s^{\eta} \sum_t P_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}} = \frac{E(h)^{\eta} C(h)^{\varepsilon(s)-\eta}}{P_s^{\eta}}. \tag{14}$$

From equation 13, we can further have:

$$\frac{\partial \ln C(h)}{\partial \ln E(h)} = \frac{\partial C(h)}{\partial E(h)} \frac{E(h)}{C(h)} = \frac{\sum_t P_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}}{\sum_t \frac{\varepsilon(s)-\eta}{1-\eta} P_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}},$$

and from equation 14 we can have:

$$\begin{aligned}\frac{\partial \ln C_s(h)}{\partial \ln E(h)} &= (\epsilon(s)-\eta)\frac{\partial \ln C(h)}{\partial \ln E(h)} + \eta \frac{\partial \ln E(h)}{\partial \ln E(h)} \\ &= (\varepsilon(s)-\eta) \frac{\sum_t P_t^{1-\eta} C(h)^{\epsilon(t)-\eta}}{\sum_t \frac{\epsilon(t)-\eta}{1-\eta} P_t^{1-\eta} C(h)^{\epsilon(t)-\eta}} + \eta\end{aligned}$$

Therefore the elasticity of human capital against sectoral consumption can be obtained

as:

$$\frac{\partial \ln C_s(h)}{\partial \ln h} = \frac{\partial \ln C_s(h)}{\partial \ln E(h)} \frac{\partial \ln E(h)}{\partial \ln h}$$
$$= \left\{ (\varepsilon(s) - \eta) \frac{\sum_t P_t^{1-\eta} C(h)^{\epsilon(t)-\eta}}{\sum_t \frac{\epsilon(t)-\eta}{1-\eta} P_t^{1-\eta} C(h)^{\epsilon(t)-\eta}} + \eta \right\} * \left\{ 1 - \frac{\ell(h)}{1-\ell(h)} \frac{\partial \ln \ell(h)}{\partial \ln h} \right\}.$$

A.3 Solving for Net Exports

In a two-country setting, in this section, we intend to solve each country's net export. Since

$$D_s^{ij}(v) = N^j \frac{\int_{\underline{h}}^{\overline{h}} E^j(h)^\eta C^j(h)^{\varepsilon(s)-\eta} dG(h)}{P_s^{j\eta-\sigma}} * p_s^{ij}(v)^{-\sigma}.$$

We define b_s^j as follows:

$$b_s^j = N^j \frac{\int_{\underline{h}}^{\overline{h}} E^j(h)^\eta C^j(h)^{\varepsilon(s)-\eta} dG(h)}{P_s^{j\eta-\sigma}}.$$

From the expression of expenditure share 10, we can have:

$$b_s^j = m_s^j P_s^{j\sigma-1} w^j L_j^e. \tag{15}$$

We also denote f_s^j to be the fraction of firms in sector s in country j:

$$f_s^j = \frac{V_s^j}{\sum_t V_t^j}.$$

The net export in country i of sector s specified in equation 8 can be expressed as:

$$NX_s^i = f_s^i b_s^j \rho \left(w^i \psi_s^i \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \sum_t V_t^i - f_s^j b_s^i \rho \left(w^j \psi_s^j \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \sum_t V_t^j. \tag{16}$$

Zero profit condition implies the quantity of goods produced by each firm in country i sector s should equals to $\frac{(\sigma-1)\phi_s^i}{\psi_s^i}$, so

$$\sum_j \tau_{ij} D_s^{ij}(v) = \sum_j \tau_{ij} b_s^j p_s^{ij}(v)^{-\sigma} = \frac{(\sigma-1)\phi_s^i}{\psi_s^i},$$

then we can obtain the expression for b_s^i as follows:

$$b_s^1 = \frac{\frac{(\sigma-1)\phi_s^1}{\psi_s^1}\left(w^1\psi_s^1\frac{\sigma}{\sigma-1}\right)^\sigma - \frac{(\sigma-1)\phi_s^2}{\psi_s^2}\left(w^2\psi_s^2\frac{\sigma}{\sigma-1}\right)^\sigma\rho}{1-\rho^2}, \tag{17}$$

$$b_s^2 = \frac{\frac{(\sigma-1)\phi_s^2}{\psi_s^2}\left(w^2\psi_s^2\frac{\sigma}{\sigma-1}\right)^\sigma - \frac{(\sigma-1)\phi_s^1}{\psi_s^1}\left(w^1\psi_s^1\frac{\sigma}{\sigma-1}\right)^\sigma\rho}{1-\rho^2}. \tag{18}$$

Given the definiton of f_s^i, the number of firms in sector s of country i equals $f_s^i\sum_t V_t^i$. By equating equation (15) and (17)-(18), we can get:

$$P_{1s}^{1-\sigma} = \frac{(1-\rho^2)\, m_s^1 w^1 L_1^e}{\frac{(\sigma-1)\phi_s^1}{\psi_s^1}\left(w^1\psi_s^1\frac{\sigma}{\sigma-1}\right)^\sigma - \frac{(\sigma-1)\phi_s^2}{\psi_s^2}\left(w^2\psi_s^2\frac{\sigma}{\sigma-1}\right)^\sigma\rho},$$

$$P_{2s}^{1-\sigma} = \frac{(1-\rho^2)\, m_s^2 w^2 L_2^e}{\frac{(\sigma-1)\phi_s^2}{\psi_s^2}\left(w^2\psi_s^2\frac{\sigma}{\sigma-1}\right)^\sigma - \frac{(\sigma-1)\phi_s^1}{\psi_s^1}\left(w^1\psi_s^1\frac{\sigma}{\sigma-1}\right)^\sigma\rho}.$$

In addition, the sector level price can also be expressed in the following way:

$$P_{1s}^{1-\sigma} = p_s^{11}(v)^{1-\sigma} f_s^1 \sum_t V_t^1 + p_s^{21}(v)^{1-\sigma} f_s^2 \sum_t V_t^2$$

$$P_{2s}^{1-\sigma} = p_s^{12}(v)^{1-\sigma} f_s^1 \sum_t V_t^1 + p_s^{22}(v)^{1-\sigma} f_s^2 \sum_t V_t^2$$

Equating the above two different expressions of sector level prices gives:

$$f_s^1 = \frac{\frac{m_s^1 w^1 L_1^e}{\frac{\phi_s^1}{\psi_s^1}\left(\psi_s^1\right)^\sigma (w^1)^\sigma - \frac{\phi_s^2}{\psi_s^2}\left(\psi_s^2\right)^\sigma \rho (w^2)^\sigma} - \frac{\rho m_s^2 w^2 L_2^e}{\frac{\phi_s^2}{\psi_s^2}\left(\psi_s^2\right)^\sigma (w^2)^\sigma - \frac{\phi_s^1}{\psi_s^1}\left(\psi_s^1\right)^\sigma \rho (w^1)^\sigma}}{\sigma\left(\psi_s^1\right)^{1-\sigma}(w^1)^{1-\sigma}\sum_t V_t^1}, \tag{19}$$

$$f_s^2 = \frac{\frac{m_s^2 w^2 L_2^e}{\frac{\phi_s^2}{\psi_s^2}\left(\psi_s^2\right)^\sigma (w^2)^\sigma - \frac{\phi_s^1}{\psi_s^1}\left(\psi_s^1\right)^\sigma \rho (w^1)^\sigma} - \frac{\rho m_s^1 w^1 L_1^e}{\frac{\phi_s^1}{\psi_s^1}\left(\psi_s^1\right)^\sigma (w^1)^\sigma - \frac{\phi_s^2}{\psi_s^2}\left(\psi_s^2\right)^\sigma \rho (w^2)^\sigma}}{\sigma\left(\psi_s^2\right)^{1-\sigma}(w^2)^{1-\sigma}\sum_t V_t^2}. \tag{20}$$

By substituting the expression of f_s^j from equation (19)-(20), and the expression of b_s^j from equation (17)-(18) into the expression of net export in equation 16, we can finally get the expression of net export in country i as:

$$NX_s^i = \rho L_j^e w^j \left\{ \frac{\frac{\phi_s^j}{\psi_s^j}\left(\psi_s^j\right)^\sigma m_s^i \omega \frac{L_i^e}{L_j^e}}{\frac{\phi_s^i}{\psi_s^i}\left(\psi_s^i\right)^\sigma \omega^\sigma - \frac{\phi_s^j}{\psi_s^j}\left(\psi_s^j\right)^\sigma \rho} - \frac{\frac{\phi_s^i}{\psi_s^i}\left(\psi_s^i\right)^\sigma \omega^\sigma m_s^j}{\frac{\phi_s^j}{\psi_s^j}\left(\psi_s^j\right)^\sigma - \frac{\phi_s^i}{\psi_s^i}\left(\psi_s^i\right)^\sigma \rho\omega^\sigma} \right\}.$$

Since the two countries are only different in the human capital distribution, so we can simplify the above net export expression as the following:

$$NX_s^i = \rho L_j^e w^j \left\{ \frac{m_s^i \omega}{\omega^\sigma - \rho} \frac{L_i^e}{L_j^e} - \frac{\omega^\sigma m_s^j}{1 - \rho\omega^\sigma} \right\}.$$

A.4 Proofs

Proof for Proposition 1

Proof: In order to prove the elasticity of human capital h with respect to labor supply or expenditure is less than 1, we first calculate the elasticities. By rearranging equation 2, we can have another equation about the relationship between $\ell(h)$ and $C(h)$.

$$\ell(h) = 1 - \frac{\sum_t P_t^{1-\eta} C(h)^{\epsilon(t)-\eta}}{wh} \tag{21}$$

Let $\xi(\ell) = \frac{d\ell(h)}{dh}\frac{h}{1-\ell(h)}$, so the elasticity of human capital against expenditure $E(h)$ equals to $1 - \xi(\ell)$. Similarly we also define the elasticity of human capital against consumption as $\xi(C) = \frac{dC(h)}{dh}\frac{h}{C(h)}$. Using equation 5 and equation 21, after some rearrangement we have:

$$\begin{aligned}
\xi(\ell) &= \frac{\xi(C)}{\eta - 1} \frac{\left(\sum_t (\varepsilon(t) - \eta) P_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}\right)}{\sum_t P_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}} + 1 \\
\xi(\ell) &= \frac{-\xi(c)}{\gamma \ell(h)^{-1} + (1-\gamma)} \left\{ 1 - \gamma + \frac{\sum_t \varepsilon(t) P_t^{1-\eta} C(h)^{\varepsilon(t)}}{\sum P_t^{1-\eta} C(h)^{\varepsilon(t)}} - \frac{\sum_t [\eta - \varepsilon(t)] \varepsilon(t) C(h)^{\varepsilon(t)} P_t^{1-\eta}}{\sum_t [\eta - \varepsilon(t)] C(h)^{\varepsilon(t)} P_t^{1-\eta}} \right\}.
\end{aligned}$$

We let

$$\begin{aligned}\Psi &= \frac{1}{(1-\eta)}\frac{\left(\sum_t (\varepsilon(t)-\eta) P_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}\right)}{\sum_t P_t^{1-\eta} C(h)^{\varepsilon(t)-\eta}} \\ &\quad * \frac{\left(\gamma \ell(h)^{-1} + (1-\gamma)\right)}{\left\{1-\gamma+\frac{\sum_t \varepsilon(t) P_t^{1-\eta} C(h)^{\varepsilon(t)}}{\sum_t P_t^{1-\eta} C(h)^{\varepsilon(t)}} - \frac{\sum_t [\eta-\varepsilon(t)]\varepsilon(t) C(h)^{\varepsilon(t)}\left(P_t^j\right)^{1-\eta}}{\sum_t [\eta-\varepsilon(t)] C(h)^{\varepsilon(t)}\left(P_t^j\right)^{1-\eta}}\right\}},\end{aligned}$$

then combining the above two equations about the elasticity of human capital h against labor supply $\xi(\ell)$ gives

$$(1-\Psi)\,\xi(\ell) = 1.$$

So we need the condition $\Psi < 0$ in order to have $0 < \xi(\ell) < 1$, which means the elasticity of human capital against expenditure $1-\xi(\ell)$ is in the range of $(0,1)$.

So we need to prove the following inequality holds:

$$1-\gamma+\frac{\sum_t \varepsilon(t) P_t^{1-\eta} C(h)^{\varepsilon(t)}}{\sum_t P_t^{1-\eta} C(h)^{\varepsilon(t)}} - \frac{\sum_t [\eta-\varepsilon(t)]\,\varepsilon(t)\, C(h)^{\varepsilon(t)}\left(P_t^j\right)^{1-\eta}}{\sum_t [\eta-\varepsilon(t)]\, C(h)^{\varepsilon(t)}\left(P_t^j\right)^{1-\eta}} < 0$$

From the left hand side of the above inequality, we have

$$\begin{aligned}& 1-\gamma+\frac{\sum_t \varepsilon(t) P_t^{1-\eta} C(h)^{\varepsilon(t)}}{\sum_t P_t^{1-\eta} C(h)^{\varepsilon(t)}} - \frac{\sum_t [\eta-\varepsilon(t)]\,\varepsilon(t)\, C(h)^{\varepsilon(t)}\left(P_t^j\right)^{1-\eta}}{\sum_t [\eta-\varepsilon(t)]\, C(h)^{\varepsilon(t)}\left(P_t^j\right)^{1-\eta}} \\ <\ & 1-\gamma+\left[1-\frac{\eta-\varepsilon(S)}{\eta-\varepsilon(1)}\right]\frac{\sum_t \varepsilon(t) P_t^{1-\eta} C(h)^{\varepsilon(t)}}{\sum_t P_t^{1-\eta} C(h)^{\varepsilon(t)}} \\ =\ & 1-\gamma+\frac{\varepsilon(1)+\varepsilon(S)}{\eta-\varepsilon(1)}\frac{\sum_t \varepsilon(t) P_t^{1-\eta} C(h)^{\varepsilon(t)}}{\sum_t P_t^{1-\eta} C(h)^{\varepsilon(t)}} \\ <\ & 1-\gamma+\frac{\varepsilon(1)+\varepsilon(S)}{\eta-\varepsilon(1)}\varepsilon(S) \\ <\ & 1-\gamma+\frac{2\varepsilon(S)^2}{\eta}\end{aligned}$$

so we need

$$1-\gamma+\frac{2\varepsilon(S)^2}{\eta} < 0$$

which can be further simplified to

$$\frac{\varepsilon\left(S\right)^{2}}{\eta}<\gamma-1$$